basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 6 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. SIX diagram sheets for QUESTIONS 2.2.1, 2.2.2, 7.4, 8.1, 8.2, 8.3, 9.1, 9.2 and 10 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

8. Diagrams are NOT necessarily drawn to scale.

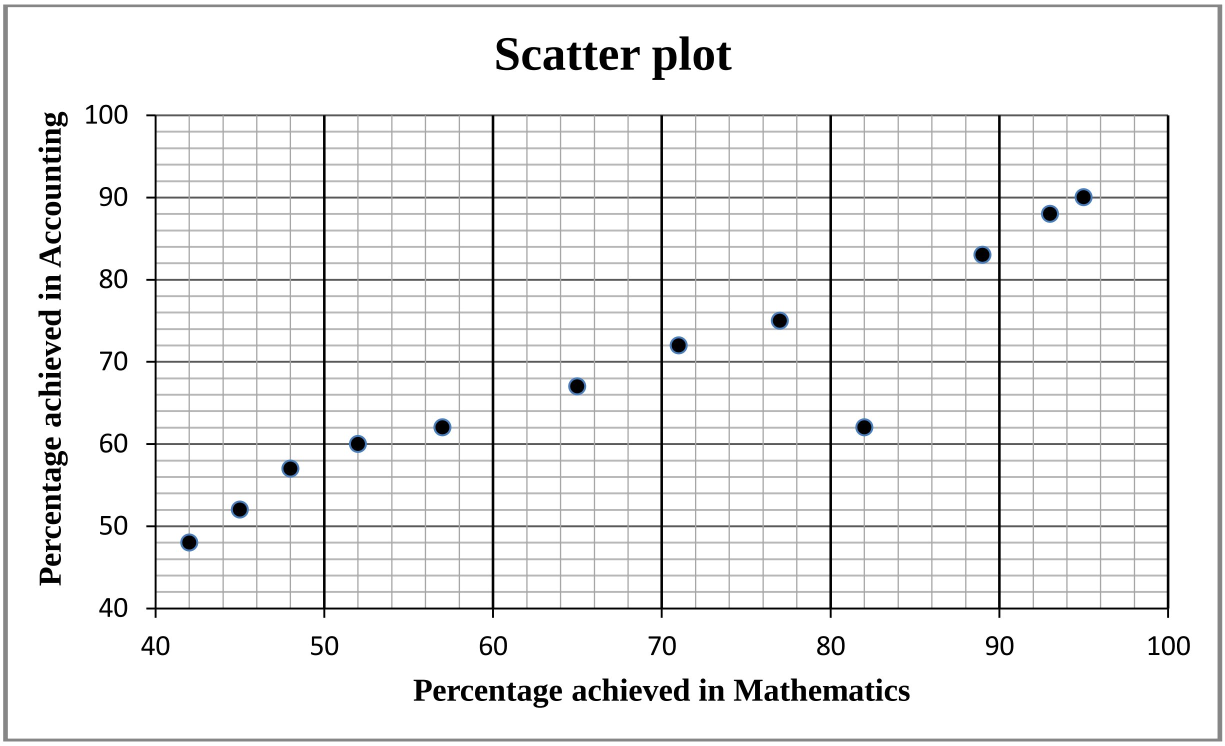
9. Number the answers correctly according to the numbering system used in this question paper.

10. Write neatly and legibly.

QUESTION 1

At a certain school, only 12 candidates take Mathematics and Accounting. The marks, as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62



1.1 Calculate the mean percentage of the Mathematics data. (2)

1.2 Calculate the standard deviation of the Mathematics data. (1)

1.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean. (3)

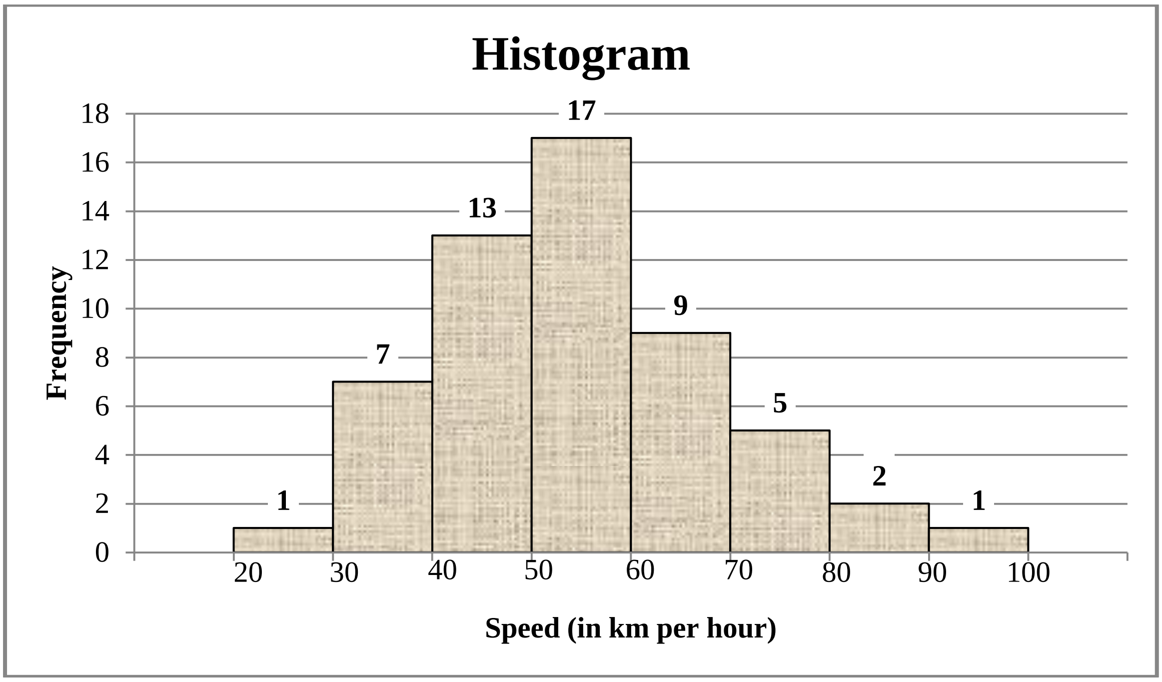
1.4 Calculate an equation for the least squares regression line (line of best fit) for the data. (3)

1.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in QUESTION 1.4. (Round off your answer to the NEAREST INTEGER.) (2)

1.6 Use the scatter plot and identify any outlier(s) in the data. (1)
[12]

QUESTION 2

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



2.1 Identify the modal class of the data. (1)

2.2 Use the histogram to:

2.2.1 Complete the cumulative frequency column in the table on DIAGRAM SHEET 1 (2)

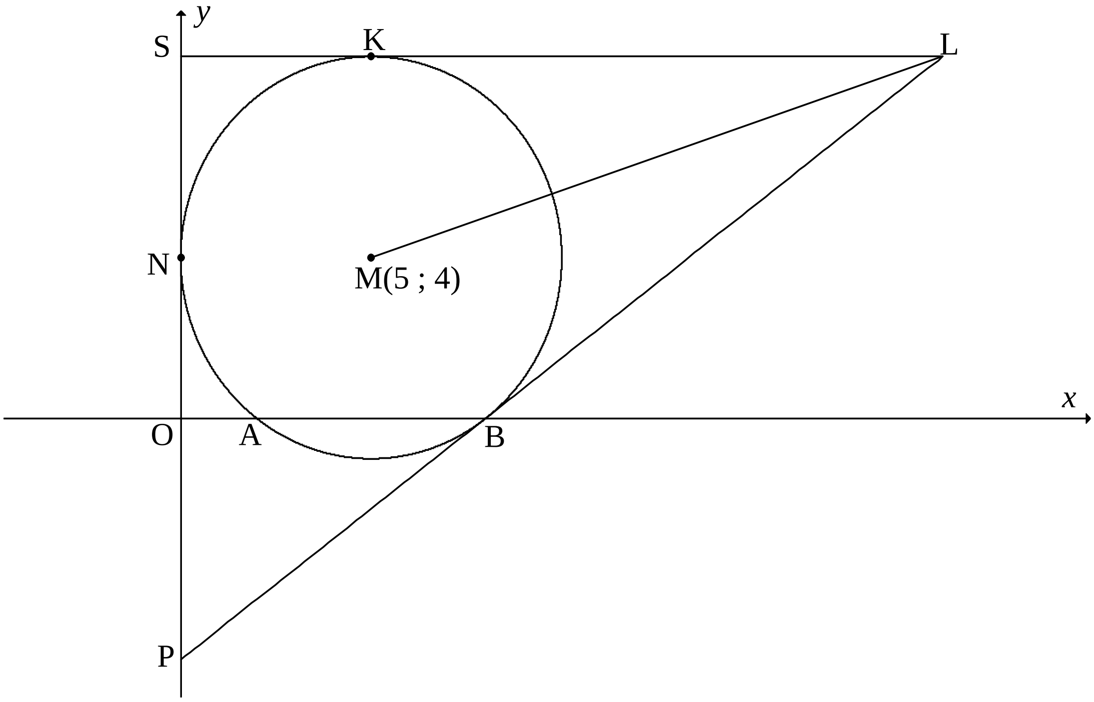
2.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid on DIAGRAM SHEET 1 (3)

2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2)

[8]

QUESTION 3

In the diagram below, a circle with centre M(5 ; 4) touches the *y*-axis at N and intersects the *x*-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the *x*-axis and P and S are points on the *y*-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y = mx + c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

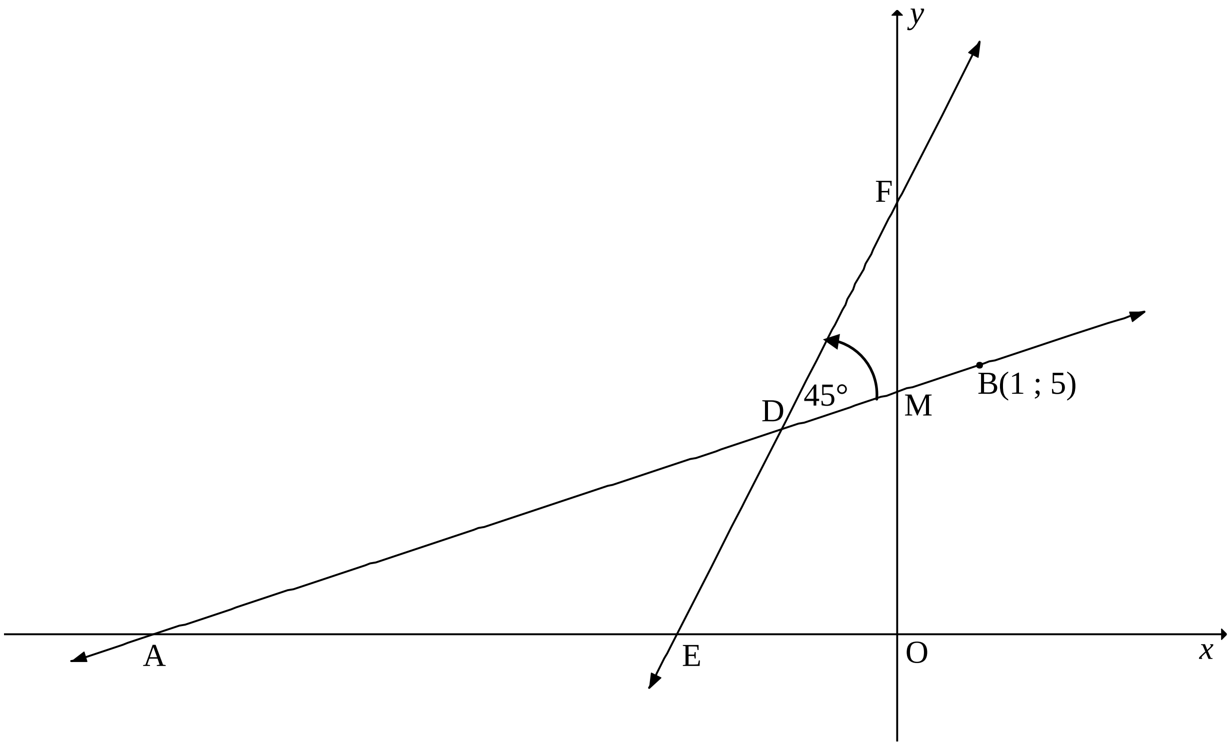
3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

QUESTION 4

In the diagram below, E and F respectively are the *x*- and *y*-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has *x*- and *y*-intercepts A and M respectively.



4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

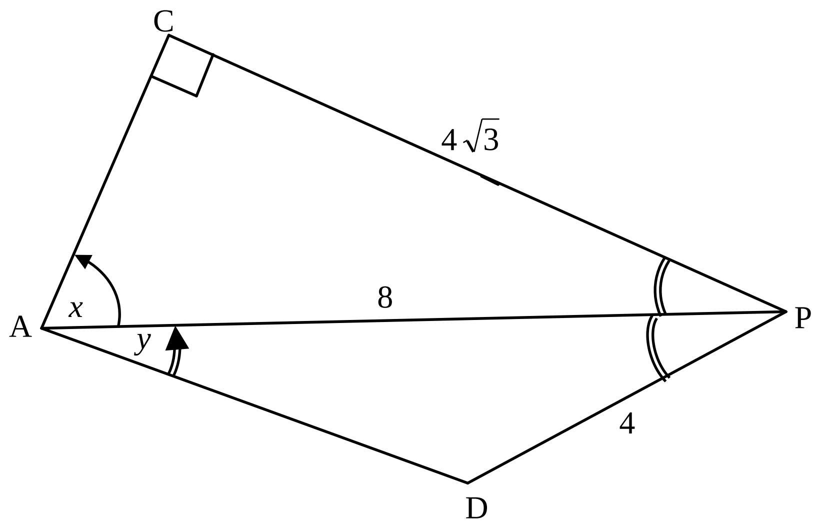
4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

4.5 Calculate the area of quadrilateral DMOE. (6)

[19]

QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



5.1 Show, by calculation, that $x = 60°$. (2)

5.2 Calculate the length of AD. (4)

5.3 Determine y. (3)
[9]

QUESTION 6

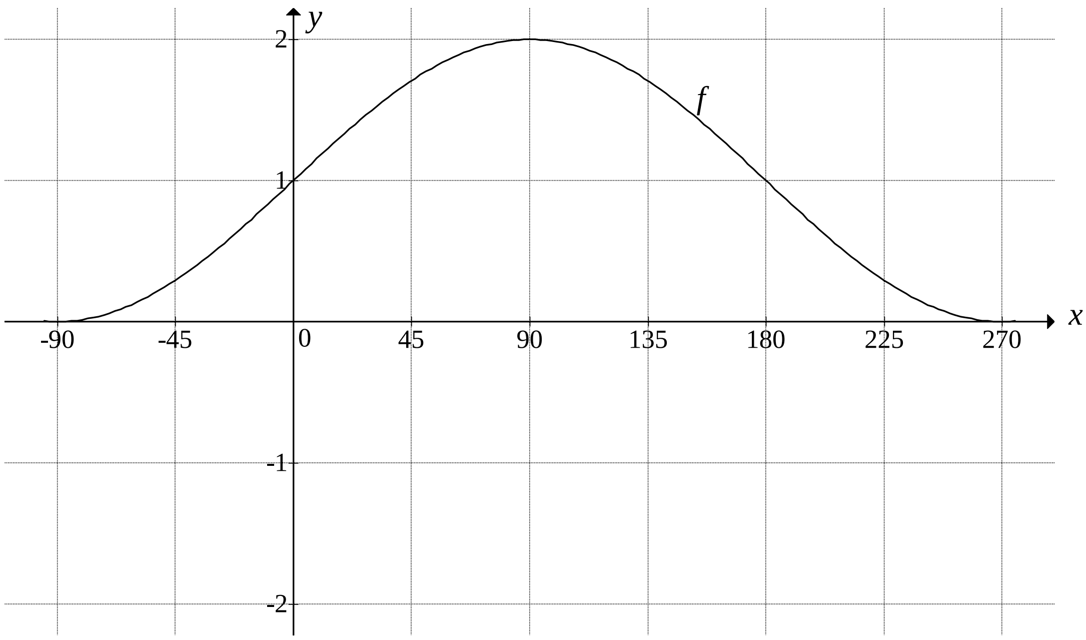
6.1 Prove the identity: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

6.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$. (4)
[12]

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

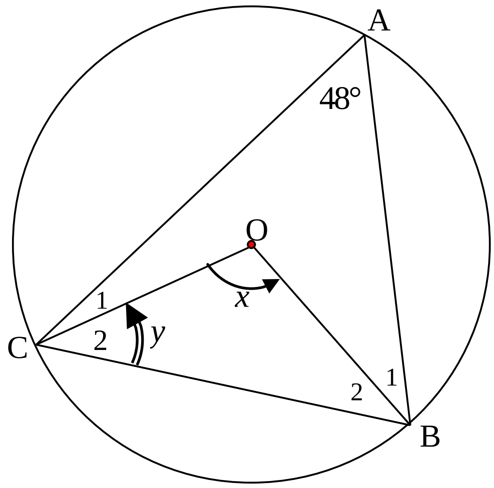
Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)

[19]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

8.1 In the diagram, O is the centre of the circle passing through A, B and C. $C\hat{A}B = 48°$, $C\hat{O}B = x$ and $\hat{C}_2 = y$.

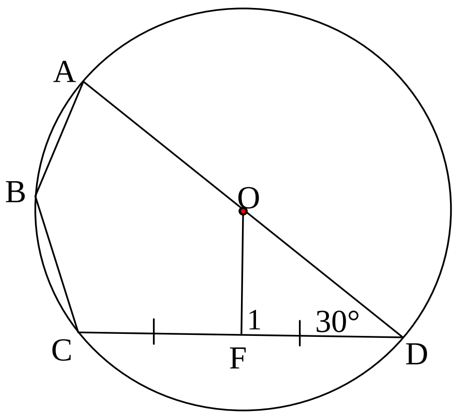


Determine, with reasons, the size of:

8.1.1 x (2)

8.1.2 y (2)

8.2 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. $O\hat{D}F = 30°$ and OF are joined.

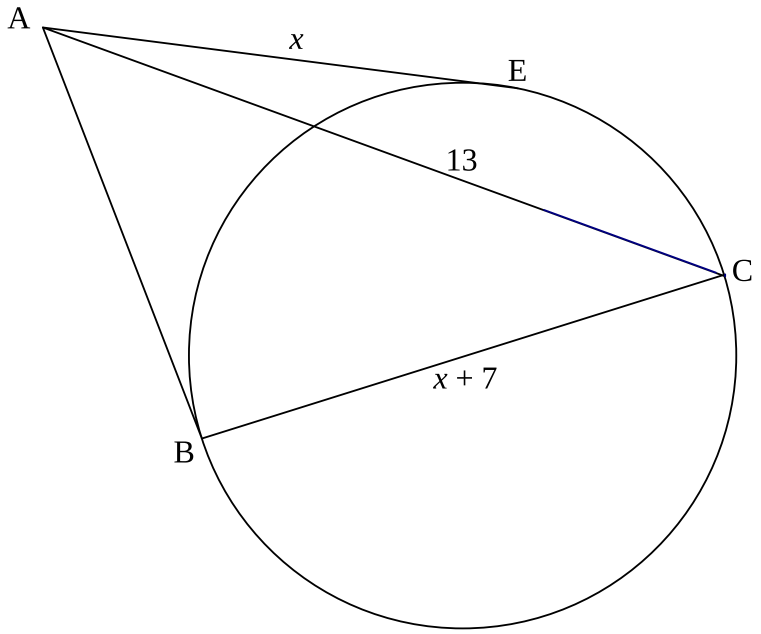


Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $A\hat{B}C$ (2)

8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. AC = 13, AE = x and BC = x + 7.



8.3.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 3.

	Statement	Reason
(a)	$A\hat{B}C = 90°$	
(b)	$AB = x$	

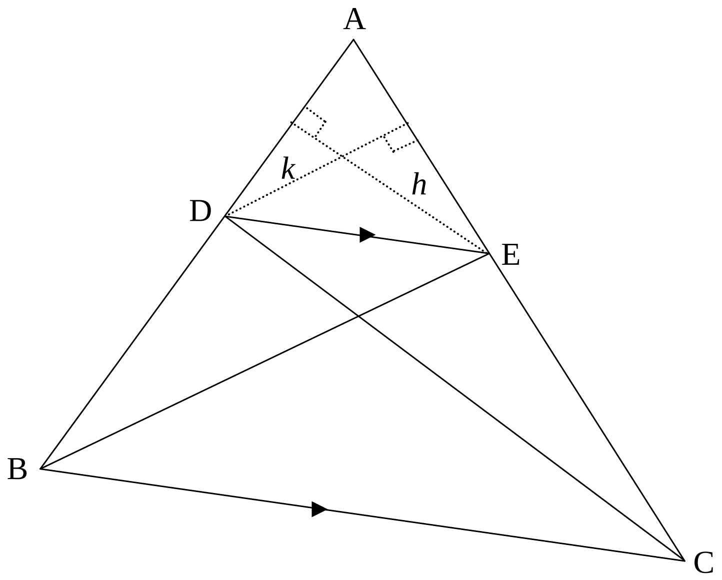
(2)

8.3.2 Calculate the length of AB. (4)

[14]

QUESTION 9

9.1 In the diagram, points D and E lie on sides AB and AC of ΔABC respectively such that DE | | BC. DC and BE are joined.



9.1.1 Explain why the areas of ΔDEB and ΔDEC are equal. (1)

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line DE | | BC then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \text{........}$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \text{......................} = \frac{AE}{EC}$$

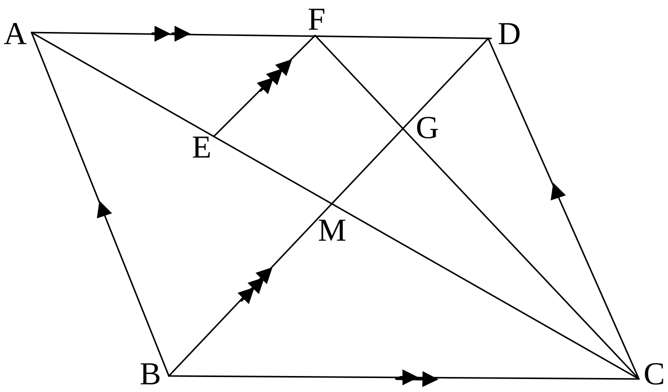
But area ΔDEB = ... (reason: ....)

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \text{...............................}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF : FD = 4 : 3. E is a point on AM such that EF || BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

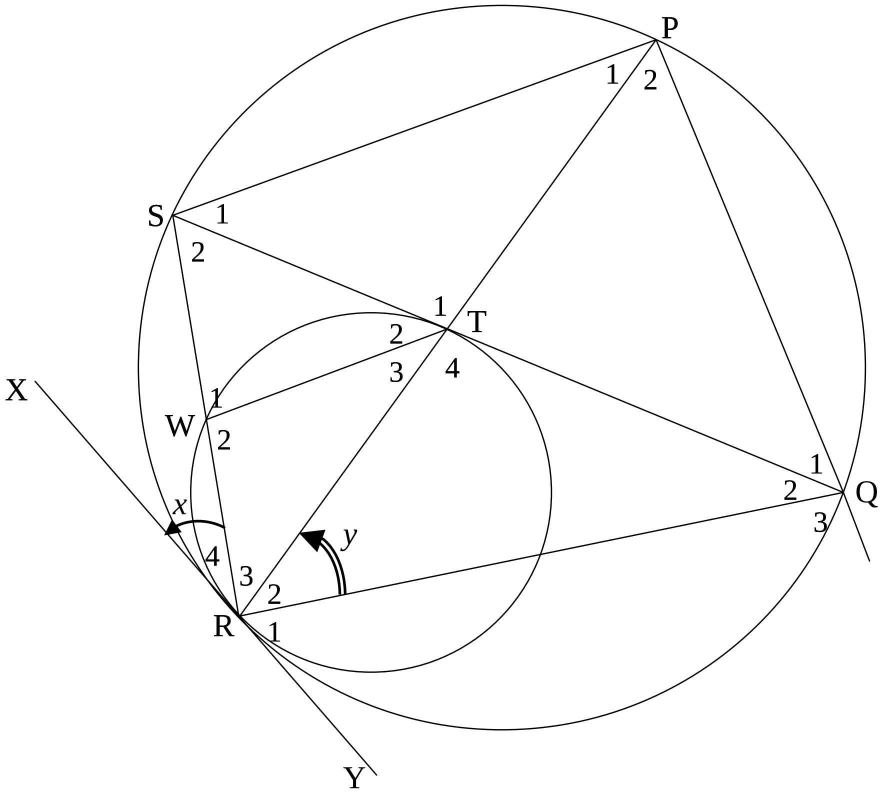
9.2.3 $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4)

[16]

QUESTION 10

The two circles in the diagram have a common tangent XRY at R. W is any point on the small circle. The straight line RWS meets the large circle at S. The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$



10.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 6.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT | | SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

10.2 Prove that $\text{RT} = \dfrac{\text{WR.RP}}{\text{RS}}$ (2)

10.3 Identify, with reasons, another TWO angles equal to y. (4)

10.4 Prove that $\hat{\text{Q}}_3 = \hat{\text{W}}_2$. (3)

10.5 Prove that $\Delta\text{RTS} \mid\mid\mid \Delta\text{RQP}$. (3)

10.6 Hence, prove that $\dfrac{\text{WR}}{\text{RQ}} = \dfrac{\text{RS}^2}{\text{RP}^2}$. (3)

[20]

TOTAL: 150

CENTRE NUMBER:

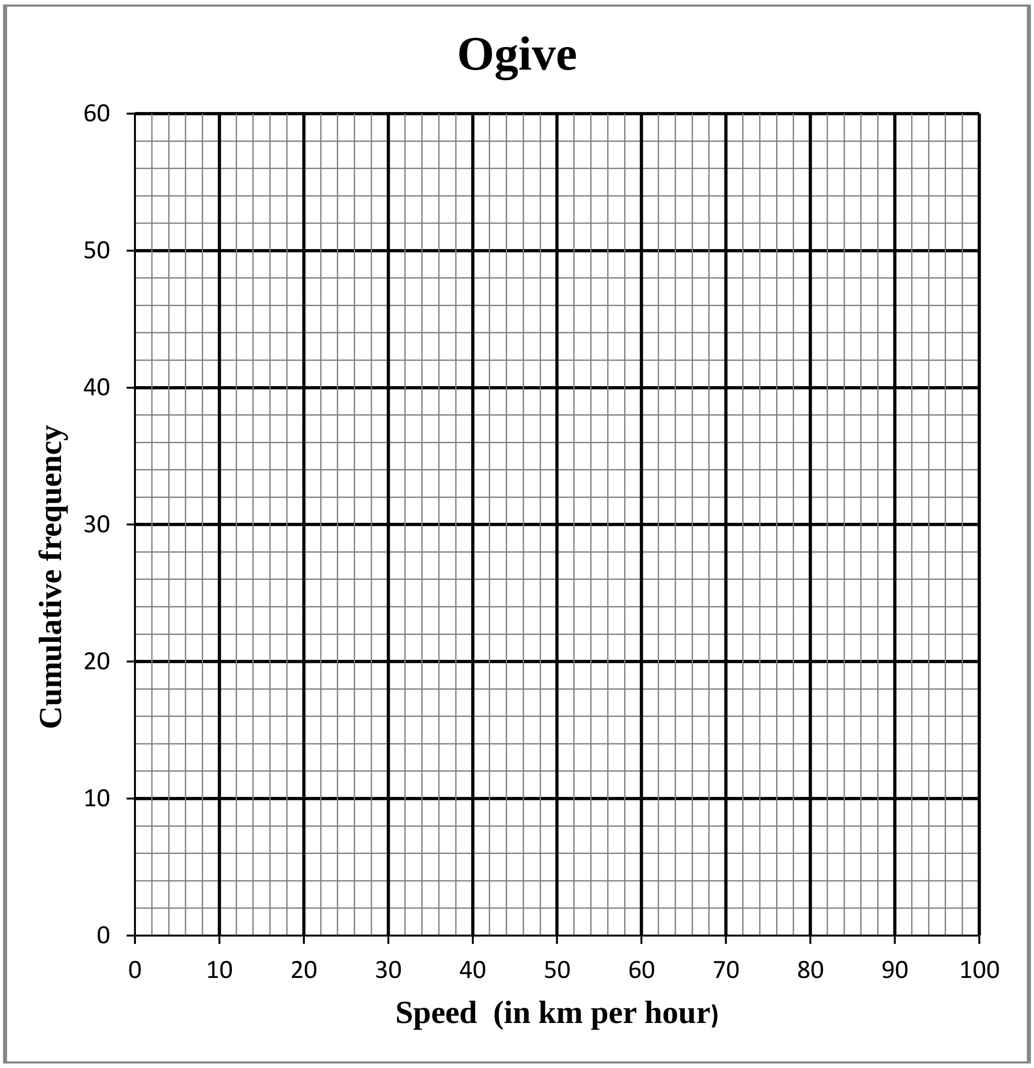
EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 2.2.1

Class	Frequency	Cumulative frequency
$20 < x \leq 30$	1	
$30 < x \leq 40$	7	
$40 < x \leq 50$	13	
$50 < x \leq 60$	17	
$60 < x \leq 70$	9	
$70 < x \leq 80$	5	
$80 < x \leq 90$	2	
$90 < x \leq 100$	1	

QUESTION 2.2.2

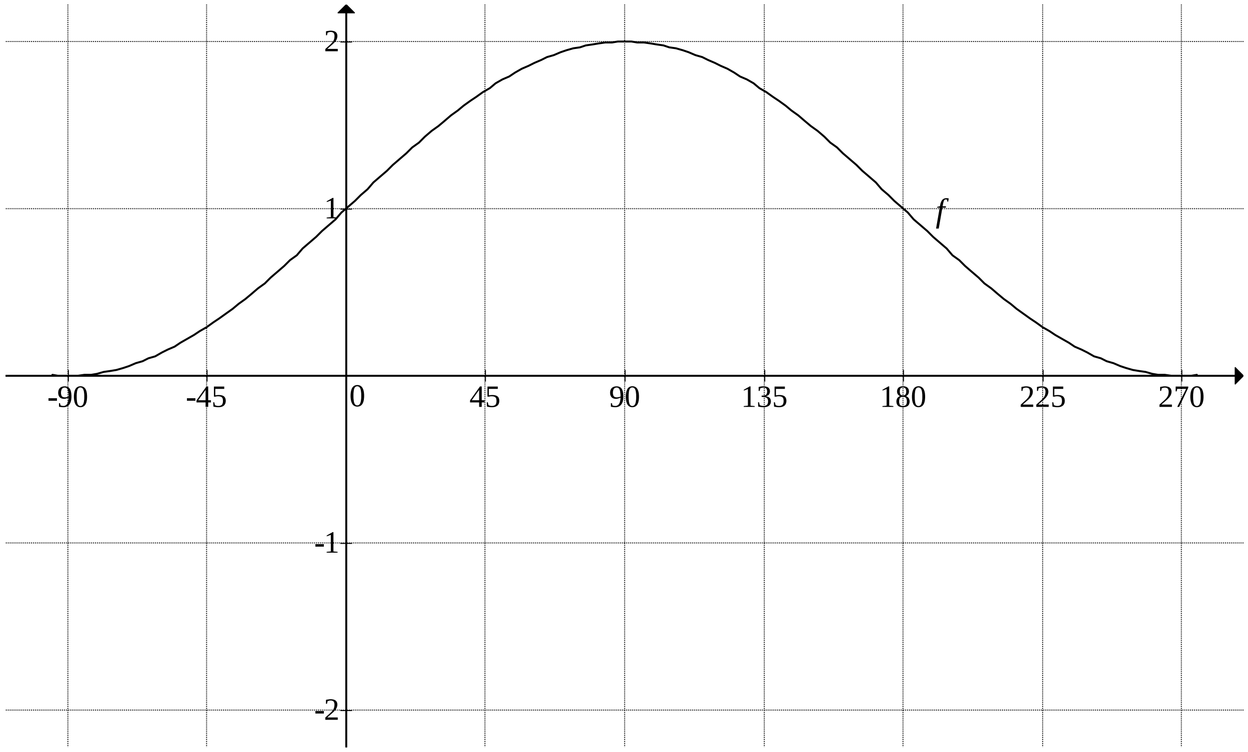


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 7.4



QUESTION 8.1

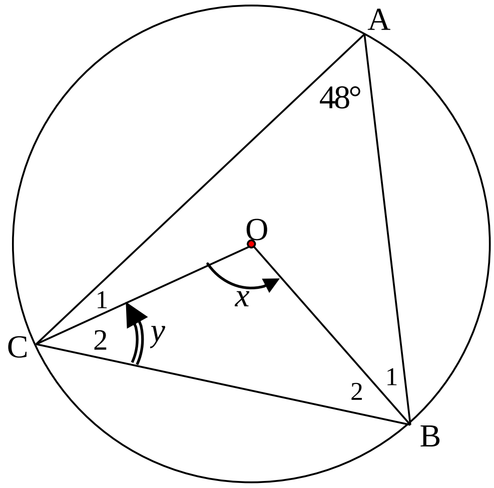


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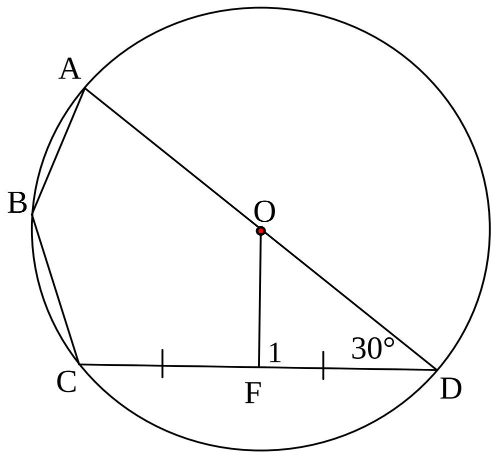
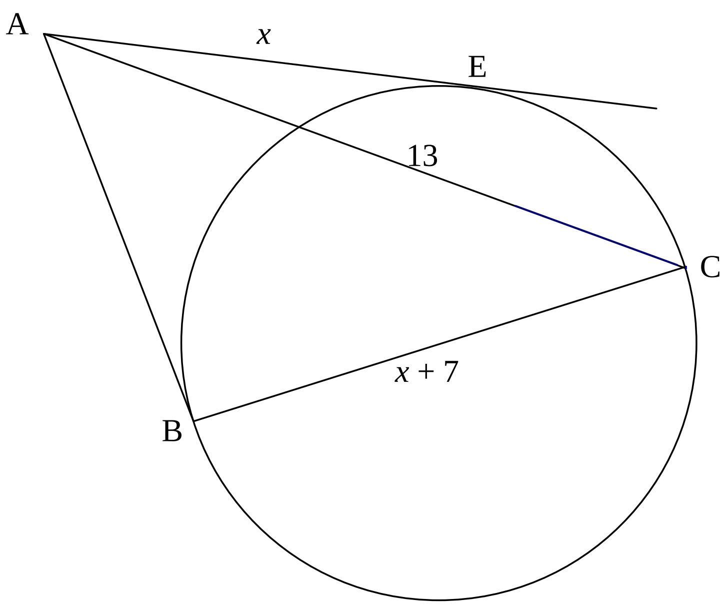
DIAGRAM SHEET 3

QUESTION 8.2



QUESTION 8.3



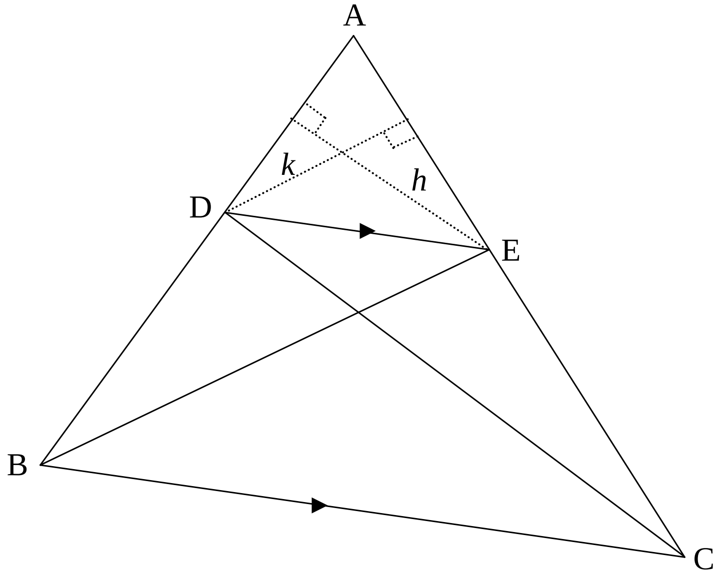
8.3.1	**Statement**	**Reason**
(a)	$A\hat{B}C = 90°$	
(b)	$AB = x$	

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 4

QUESTION 9.1



9.1.2 Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \frac{\frac{1}{2}(\text{AD})(h)}{\frac{1}{2}(\text{BD})(h)} = \text{........}$$

$$\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \text{......................} = \frac{\text{AE}}{\text{EC}}$$

But area ΔDEB = ...

(reason: ....)

$$\therefore \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \text{...............................}$$

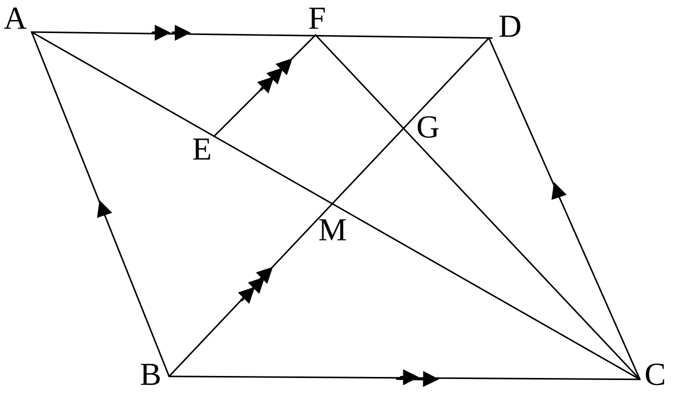
$$\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$$

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 5

QUESTION 9.2

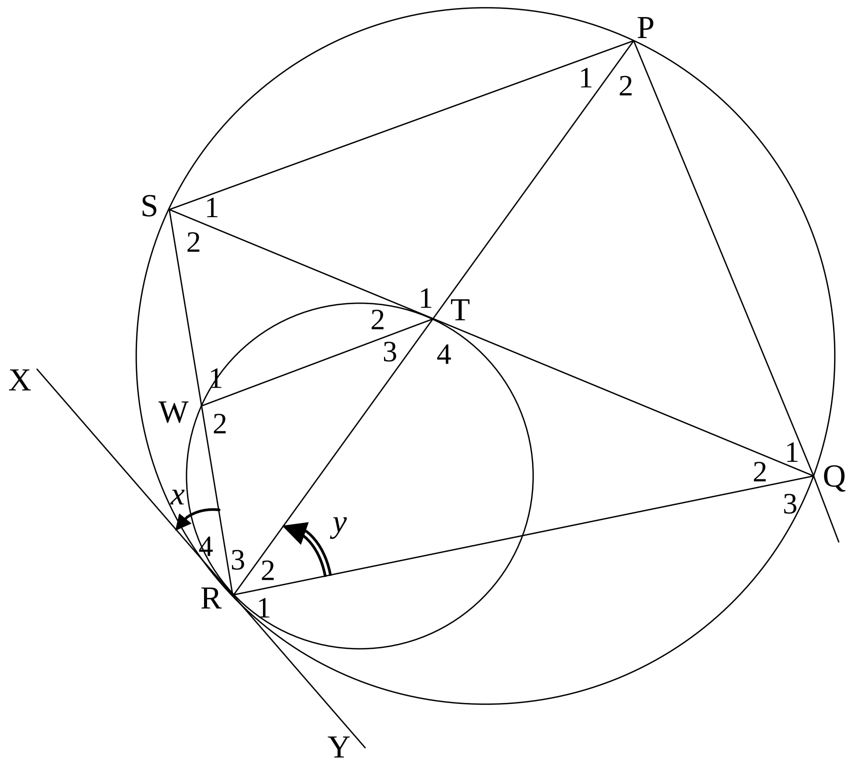


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 6

QUESTION 10



Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT | | SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$ **;** $r \neq 1$ $S_\infty = \frac{a}{1-r}$ **;** $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$