



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL SENIOR CERTIFICATE/
*NASIONALE SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

**These marking guidelines consist of 24 pages.
*Hierdie nasienriglyne bestaan uit 24 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

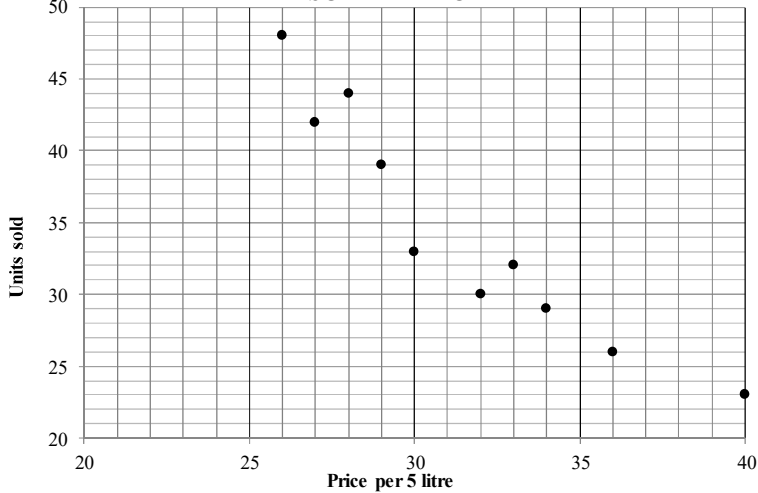
QUESTION/VRAAG 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

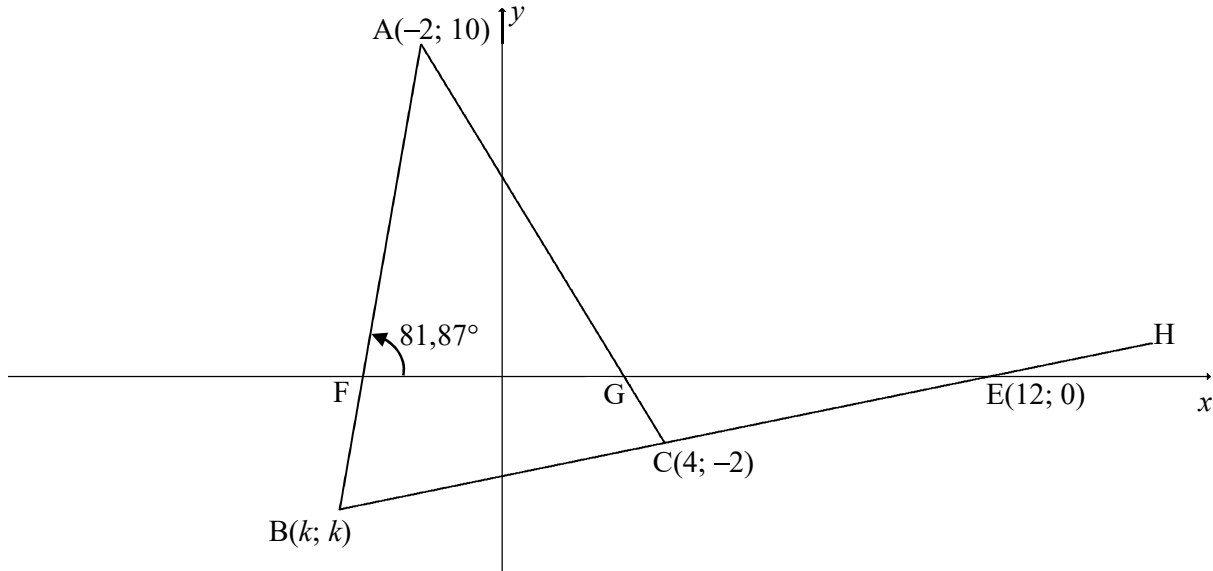
1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only: Full marks <i>Slegs antw: Volpunte</i> </div>	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10,1707 \approx 10,17$		✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32,17$ $\therefore 5$ days		✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/ <i>bestel</i> $20 \times 18 = 360$ sold/ <i>verkoop</i> Total not sold/ <i>Totaal nie verkoop nie: 36</i> OR/OF $22 - 20 = 2$ $2 \times 18 = 36$		✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/ <i>Opsie B</i> <u>Any one of the following reasons/<i>Enige een van die vlg redes:</i></u> <ul style="list-style-type: none"> • Median/<i>Mediaan</i> = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right 		✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right <i>Data is positief skeef/skeef na regs</i>		✓ answer (1)
			[10]

QUESTION/VRAAG 2

Price of milk in rands per 5-litre container (x) <i>Prys van melk in rand, per 5 liter-houer (x)</i>	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) <i>Aantal 5 liter-houers melk verkoop (y)</i>	48	30	26	44	23	32	39	29	42	33

<p>2.1</p>	<p style="text-align: center;">SCATTER PLOT</p> 	<p>1 mark: 3 to 5 points plotted correctly</p> <p>2 marks: 6 to 9 points plotted correctly</p> <p>3 marks: all points plotted correctly</p> <p style="text-align: right;">(3)</p>
<p>2.2</p>	<p>$a = 90,478... \approx 90,48$ $b = -1,773... \approx -1,77$ $\hat{y} = 90,48 - 1,77x$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p style="text-align: center;">Answer only: Full marks Slegs antw: Volpunte</p> </div>	<p>✓ a ✓ b ✓ equation</p> <p style="text-align: right;">(3)</p>
<p>2.3</p>	<p>$y = 23,069... \approx 23,07$ units/eenhede (calculator/sakrekenaar)</p> <p>OR/OF</p> <p>$y = 90,48 - 1,77(38)$ $y = 23,22$ units/eenhede</p>	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p> <p>✓ substitution ✓ answer</p> <p style="text-align: right;">(2)</p>
<p>2.4</p>	<p>$r = -0,94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold \therefore there is a good chance of the prediction being accurate./ Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan \therefore daar is 'n goeie kans dat die voorspelling akkuraat is</p>	<p>✓ value of r OR/OF strong relationship/ sterk verwantskap</p> <p>✓ accurate/akkuraat</p> <p style="text-align: right;">(2)</p>
<p>[10]</p>		

QUESTION/VRAAG 3

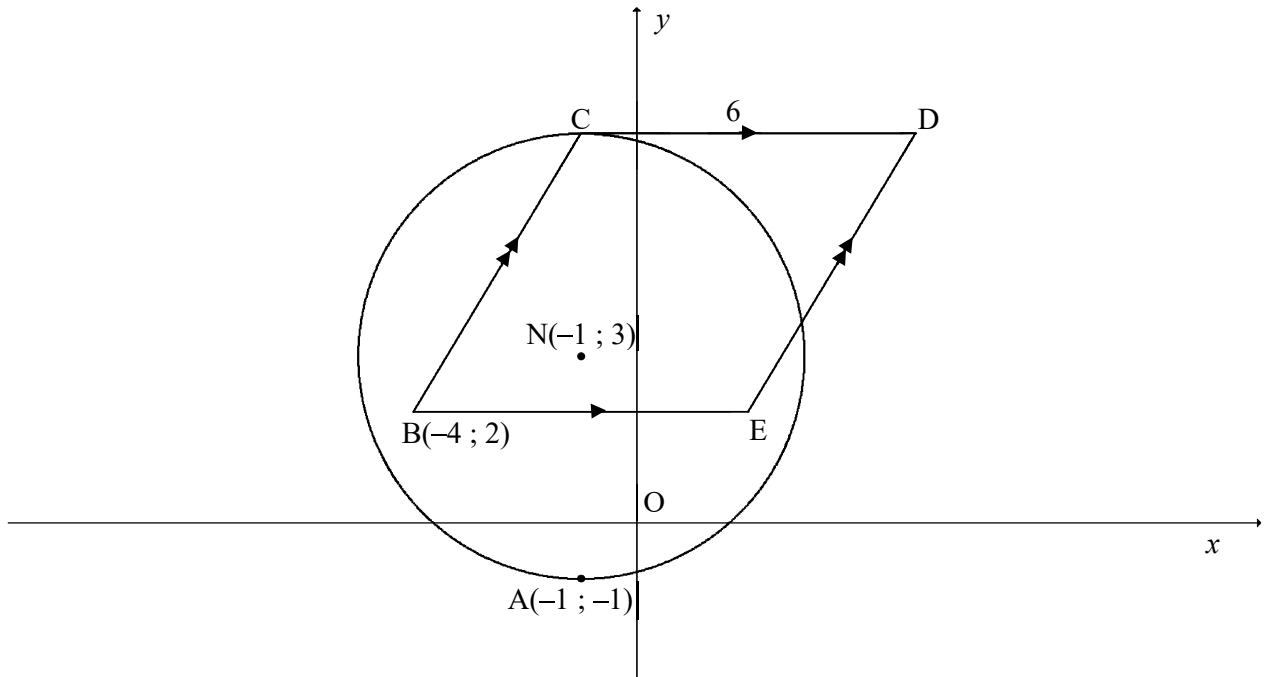


3.1.1	$m_{BE} = m_{CE} = \frac{0 - (-2)}{12 - 4} \quad \text{OR/OF} \quad m_{BE} = m_{CE} = \frac{-2 - 0}{4 - 12}$ $= \frac{1}{4} \qquad \qquad \qquad = \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$	✓ substitution ✓ answer (2)
3.2	$y = mx + c \qquad \qquad y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c \qquad \text{or} \qquad y - 0 = \frac{1}{4}(x - 12)$ $c = -3 \qquad \qquad y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ <p>OR/OF</p> $y = mx + c \qquad \qquad y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c \qquad \text{or} \qquad y - (-2) = \frac{1}{4}(x - 4)$ $c = -3 \qquad \qquad y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

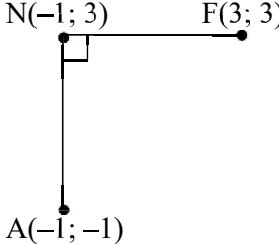
3.3.1	$y = \frac{1}{4}x - 3$ $k = \frac{1}{4}k - 3$ $\frac{3}{4}k = -3$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{BE} = \frac{1}{4}$ $\frac{0 - k}{12 - k} = \frac{1}{4}$ $-4k = 12 - k$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ $m_{AB} = \frac{10 - k}{-2 - k}$ $7(-2 - k) = 10 - k$ $-14 - 7k = 10 - k$ $-6k = 24$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> <p>EB: $y = \frac{1}{4}x - 3$ and AB: $y = 7x + 24$</p> $\frac{1}{4}x - 3 = 7x + 24$ $\frac{27}{4}x = -27$ $x = k = -4$ $\therefore B(-4; -4)$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ equating EB & AB</p> <p>✓ answer (2)</p>
-------	---	---

<p>3.3.2</p>	<p>In $\triangle AFG$:</p> $m_{AC} = \frac{10 - (-2)}{-2 - 4} = -2$ <p>$\tan \theta = m_{AC} = -2$ $\theta = 180^\circ - 63,43\dots^\circ$ $\therefore \theta = 116,57^\circ$ $\therefore \hat{A} = 116,57^\circ - 81,87^\circ$ [ext \angle of Δ] $\therefore \hat{A} = 34,70^\circ$</p> <p>OR/OF In $\triangle ABC$:</p> $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ $(2\sqrt{17})^2 = (6\sqrt{5})^2 + (10\sqrt{2})^2 - 2(6\sqrt{5})(10\sqrt{2}) \cdot \cos A$ $\cos A = \frac{(6\sqrt{5})^2 + (10\sqrt{2})^2 - (2\sqrt{17})^2}{2(6\sqrt{5})(10\sqrt{2})}$ $= 0,822\dots$ $\therefore A = 34,7^\circ$	<p>✓ $m_{AC} = -2$ ✓ $\tan \theta = -2$ ✓ $\theta = 116,57^\circ$ ✓ answer (4)</p> <p>✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)</p>
<p>3.3.3</p>	<p>$M\left(\frac{12 + (-2)}{2}; \frac{10 + (0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)</p>	<p>✓ x-value ✓ y-value (2)</p>
<p>3.4.1</p>	<p>BE = ET</p> $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $(4\sqrt{17})^2 = (\sqrt{(12-p)^2 + (0-p)^2})^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ <p>$\therefore p = 16$ or $p = -4$ (n.a.) $\therefore T(16; 16)$</p>	<p>✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)</p>
<p>3.4.2a</p>	<p>$(x-12)^2 + y^2 = (4\sqrt{17})^2 = 272$</p>	<p>✓ LHS ✓ RHS (2)</p>
<p>3.4.2b</p>	<p>$m_{\text{radius}} = \frac{1}{4}$ $m_{\text{tangent}} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$</p> <p>OR/OF</p> $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	<p>✓ m_{tangent} ✓ substitution of B ✓ equation (3)</p>
<p>[24]</p>		

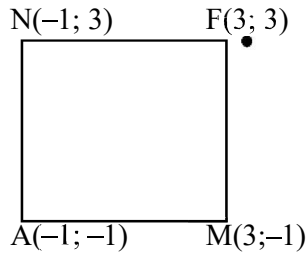
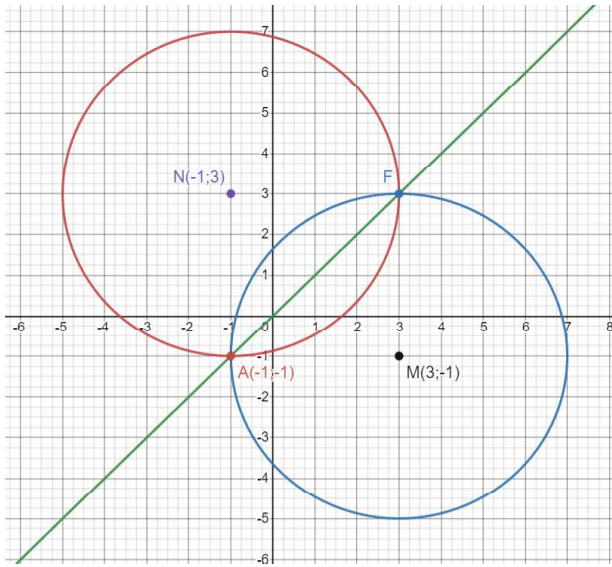
QUESTION/VRAAG 4



4.1	Radius = 4 units/eenhede	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1; 7)$	✓ x value ✓ y value (2)
4.2.2	$CD = 6$ units $\therefore D(5; 7)$	✓ x value ✓ y value (2)
4.2.3	$\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ OR/OF $\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	<p>OR/OF Let angle of inclination of BC = α $\tan \alpha = \frac{5}{3}$ $\alpha = 59,036\dots^\circ$</p> <p>$\hat{BCD} = 180^\circ - \alpha$ $\hat{BCD} = 180^\circ - 59,036\dots^\circ$ $\hat{BCD} = 120,96^\circ$</p> <p>Area $\triangle BCD = \frac{1}{2}(\sqrt{34})(6) \sin 120,96^\circ$ $= 15 \text{ units}^2$</p>	<p>✓ $\hat{BCD} = 120,96^\circ$</p> <p>✓ substitution into Area rule</p> <p>✓ answer (3)</p>
<p>4.3.1</p>	<p>M(3 ; -1) [reflection of N(-1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3 - (-1))^2 + (-1 - 3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5,66 \text{ units}$</p>	<p>✓ coordinates of M (A)</p> <p>✓ substitution of M&N</p> <p>✓ answer (3)</p>
<p>4.3.2</p>	<p>M(3 ; -1) $m_{MN} = \frac{3 - (-1)}{-1 - 3} = -1$</p> <p>MN: $-1 = -(3) + c$ or $y - 3 = -1(x + 1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$</p> <p>$x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1)</p> <p>OR/OF</p>  <p>N(-1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore F(3 ; 3)$</p> <p>midpoint $\left(\frac{-1 + 3}{2}; \frac{-1 + 3}{2}\right) = (1 ; 1)$</p>	<p>✓ equation of MN</p> <p>✓ equating AF & MN</p> <p>✓ x value ✓ y value (4)</p> <p>✓✓ coordinates of F</p> <p>✓ x value ✓ y value (4)</p>

OR/OF



NAMF is a square (NA=NF=AM=MF and $NA \perp AM$)

Midpoint NM = (1 ; 1)
= Midpoint of AF

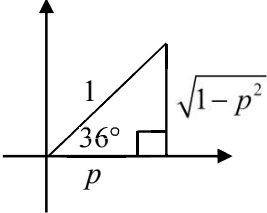
✓ NAMF = square

✓ x ✓ y of midpt NM
✓ midpt AF

(4)

[15]

QUESTION/VRAAG 5

<p>5.1</p>	$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)}$ $= \frac{\sin 40^\circ (-\sin x)}{\sin 40^\circ (-\tan x)}$ $= \frac{-\sin x}{-\tan x}$ $= \frac{\sin x}{\cos x}$ $= \cos x$	<p>✓ $\sin 40^\circ$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p>
<p>5.2</p>	$\text{LHS} = \frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)}$ $\text{LHS} = \frac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$ $\text{LHS} = \frac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \frac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \frac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ <p>$\therefore \text{LHS} = \text{RHS}$</p>	$\text{RHS} = 2\cos x - 1$ <p>✓ identity i. t. o. $\cos x$ ✓ $\cos(540^\circ - x) = -\cos x$</p> <p>✓ standard form</p> <p>✓ factors</p> <p style="text-align: right;">(4)</p>
<p>5.3.1</p>	$\sin 36^\circ = \sqrt{1 - p^2}$ $\tan 36^\circ = \frac{\sqrt{1 - p^2}}{p}$ <p>OR/OF</p> $\cos^2 36^\circ = 1 - \sin^2 36^\circ$ $\cos 36^\circ = \sqrt{1 - (1 - p^2)}$ $= p$ $\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ}$ $= \frac{\sqrt{1 - p^2}}{p}$	 <p>✓ method</p> <p>✓ value of p</p> <p>✓ answer</p> <p style="text-align: right;">(3)</p> <p>✓ method</p> <p>✓ $\cos 36^\circ = p$</p> <p>✓ answer</p> <p style="text-align: right;">(3)</p>

5.3.2	$\begin{aligned} \cos 108^\circ &= -\cos 72^\circ \\ &= -\cos (2 \times 36^\circ) \\ &= -(2 \cos^2 36^\circ - 1) \\ &= -2p^2 + 1 \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \cos 108^\circ &= -\cos 72^\circ \\ &= -\cos (2 \times 36^\circ) \\ &= -(1 - 2 \sin^2 36^\circ) \\ &= -1 + 2(\sqrt{1 - p^2})^2 \\ &= -1 + 2(1 - p^2) \\ &= -2p^2 + 1 \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \cos 108^\circ &= -\cos 72^\circ \\ &= -\cos (2 \times 36^\circ) \\ &= -(\cos^2 36^\circ - \sin^2 36^\circ) \\ &= -\left(p^2 - (\sqrt{1 - p^2})^2\right) \\ &= -(p^2 - (1 - p^2)) \\ &= -2p^2 + 1 \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \cos 108^\circ &= \cos(2 \times 54^\circ) \\ &= 2 \cos^2 54^\circ - 1 \\ &= 2(1 - p^2) - 1 \\ &= 1 - 2p^2 \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \cos 108^\circ &= \cos(72^\circ + 36^\circ) \\ &= \cos 72^\circ \cos 36^\circ - \sin 72^\circ \sin 36^\circ \\ &= (2 \cos^2 36^\circ - 1) \cos 36^\circ - (2 \sin 36^\circ \cos 36^\circ) \sin 36^\circ \\ &= 2 \cos^3 36^\circ - \cos 36^\circ - 2 \cos 36^\circ \sin^2 36^\circ \\ &= 2p^3 - p - 2p(\sqrt{1 - p^2})^2 \\ &= 2p^3 - p - 2p + 2p^3 \\ &= 4p^3 - 3p \end{aligned}$	<ul style="list-style-type: none"> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) <ul style="list-style-type: none"> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) <ul style="list-style-type: none"> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) <ul style="list-style-type: none"> ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) <ul style="list-style-type: none"> ✓ expansion ✓ both double angle identities ✓ value of $\sin 36^\circ$ ✓ answer i. t. o. p (4)
[17]		

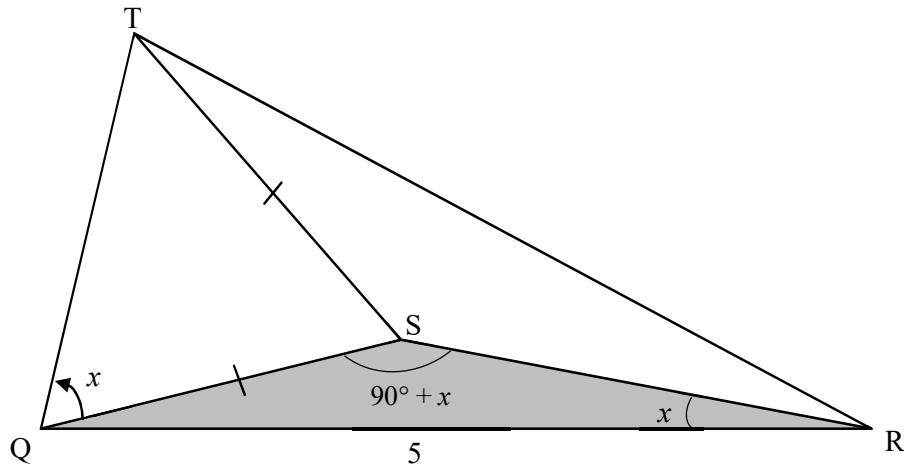
QUESTION/VRAAG 6

6.1.1	$\begin{aligned} & \cos(\alpha + \beta) \\ &= \cos(\alpha - (-\beta)) \\ &= \cos \alpha \cos(-\beta) + \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha(-\sin \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{aligned}$	<ul style="list-style-type: none"> ✓ $\cos(\alpha - (-\beta))$ ✓ expansion ✓ reduction <p style="text-align: right;">(3)</p>
6.1.2	$\begin{aligned} & 2 \cos 6x \cos 4x - \cos 10x + 2 \sin^2 x \\ &= 2 \cos 6x \cos 4x - \cos(6x + 4x) + 2 \sin^2 x \\ &= 2 \cos 6x \cos 4x - (\cos 6x \cos 4x - \sin 6x \sin 4x) + 2 \sin^2 x \\ &= \cos 6x \cos 4x + \sin 6x \sin 4x + 2 \sin^2 x \\ &= \cos 2x + 2 \sin^2 x \\ &= 1 - 2 \sin^2 x + 2 \sin^2 x \\ &= 1 \end{aligned}$	<ul style="list-style-type: none"> ✓ $\cos 10x = \cos(6x + 4x)$ ✓ expansion of $\cos(6x + 4x)$ ✓ $\cos 2x$ ✓ $1 - 2 \sin^2 x$ ✓ answer <p style="text-align: right;">(5)</p>
6.2	$\begin{aligned} \tan x &= 2 \sin 2x \\ \frac{\sin x}{\cos x} &= 2(2 \sin x \cos x) \\ \sin x &= 4 \sin x \cos^2 x \\ 4 \sin x \cos^2 x - \sin x &= 0 \\ \sin x(4 \cos^2 x - 1) &= 0 \\ \sin x = 0 & \quad \text{or} \quad \cos^2 x = \frac{1}{4} \\ & \quad \quad \quad \cos x = -\frac{1}{2} \\ x = 180^\circ + k.360^\circ; k \in Z & \quad \text{or} \quad x = 120^\circ + k.360^\circ; k \in Z \\ & \quad \quad \quad x = 240^\circ + k.360^\circ; k \in Z \\ \text{OR/OF} \\ \tan x &= 2 \sin 2x \\ \frac{\sin x}{\cos x} &= 4 \sin x \cos x \\ \sin x &= 4 \sin x \cos^2 x \\ 4 \sin x \cos^2 x - \sin x &= 0 \\ 4 \sin x(1 - \sin^2 x) - \sin x &= 0 \\ 3 \sin x - 4 \sin^3 x &= 0 \\ \sin x(3 - 4 \sin^2 x) &= 0 \\ \sin x = 0 & \quad \text{or} \quad \sin^2 x = \frac{3}{4} \\ & \quad \quad \quad \sin x = \frac{\sqrt{3}}{2} \quad \text{or} \quad \sin x = -\frac{\sqrt{3}}{2} \\ x = 180^\circ + k.360^\circ, k \in Z & \quad \text{or} \quad x = 120^\circ + k.360^\circ, k \in Z \\ & \quad \quad \quad \text{or} \quad x = 240^\circ + k.360^\circ, k \in Z \end{aligned}$	<ul style="list-style-type: none"> ✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 120^\circ$ & 240° OR/OF $x = \pm 120^\circ$ ✓ $k.360^\circ; k \in Z$ <p style="text-align: right;">(7)</p> <ul style="list-style-type: none"> ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 120^\circ$ & 240° OR/OF $x = \pm 120^\circ$ ✓ $k.360^\circ; k \in Z$ <p style="text-align: right;">(7)</p>
		[15]

QUESTION/VRAAG 7

<p>7.1</p>		<ul style="list-style-type: none"> ✓ both turning points ✓ both x intercepts (-30° & 150°) ✓ shape <p style="text-align: right;">(3)</p>
<p>7.2</p>	<p>Period = 120°</p>	<ul style="list-style-type: none"> ✓✓ answer <p style="text-align: right;">(2)</p>
<p>7.3</p>	<p>$x = -30^\circ$</p>	<ul style="list-style-type: none"> ✓ answer <p style="text-align: right;">(1)</p>
<p>7.4</p>	<p>Range of/waardeversameling van g: $y \in [-1; 1]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2}; \frac{3}{2}\right]$</p> <p>OR/OF</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $\frac{1}{2} \leq y \leq \frac{3}{2}$</p>	<ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p style="text-align: right;">(2)</p> <ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p style="text-align: right;">(2)</p>
<p>[8]</p>		

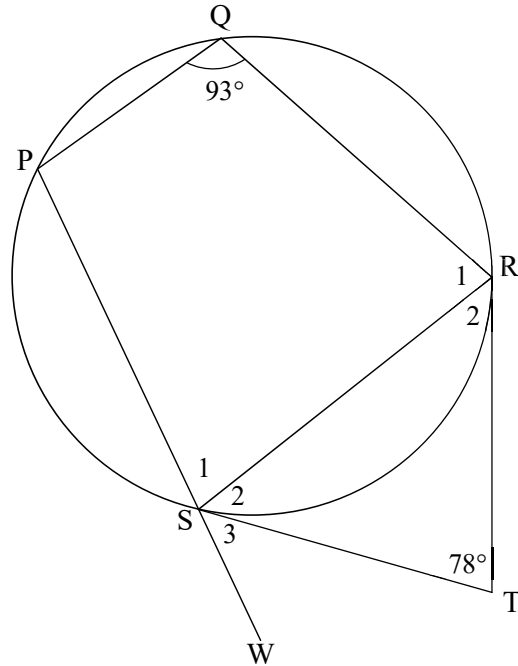
QUESTION/VRAAG 8



8.1	<p>In $\triangle SQR$:</p> $\frac{QS}{\sin x} = \frac{QR}{\sin(90^\circ + x)}$ $\frac{QS}{\sin x} = \frac{5}{\cos x}$ $QS = \frac{5 \sin x}{\cos x}$ $QS = 5 \tan x$	<p>✓ correct use of sine rule</p> <p>✓ $\sin(90^\circ + x) = \cos x$</p> <p>✓ $QS = \frac{5 \sin x}{\cos x}$</p> <p style="text-align: right;">(3)</p>
8.2	$\frac{QT}{\sin(180^\circ - 2x)} = \frac{TS}{\sin x}$ $\frac{QT}{\sin 2x} = \frac{5 \tan x}{\sin x}$ $QT = \frac{5 \tan x \sin 2x}{\sin x}$ $QT = \frac{5 \left(\frac{\sin x}{\cos x} \right) (2 \sin x \cos x)}{\sin x}$ $QT = \frac{5 \sin x (2 \sin x)}{\sin x}$ $QT = 10 \sin x$	<p>✓ correct use of sine rule</p> <p>✓ $TS = QS = 5 \tan x$</p> <p>✓ $QT = \frac{5 \tan x \sin 2x}{\sin x}$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ $\sin 2x = 2 \sin x \cos x$</p> <p style="text-align: right;">(5)</p>

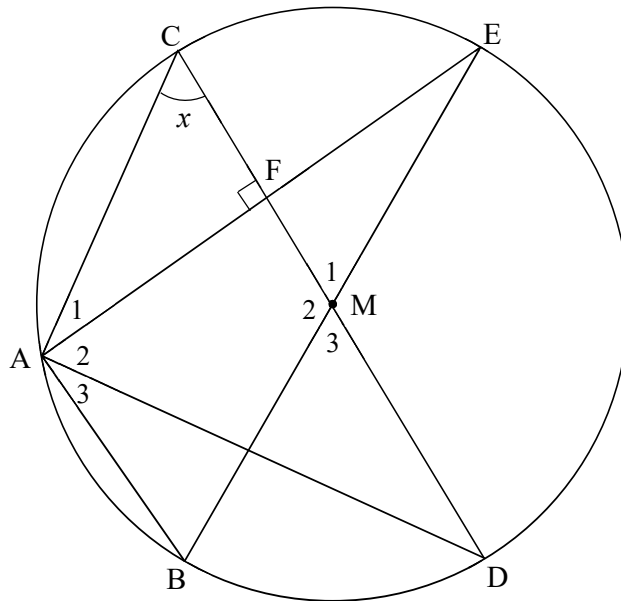
	<p>OR/OF</p> $QT^2 = QS^2 + TS^2 - 2QS.TS\cos\hat{Q}ST$ $QT^2 = (5 \tan x)^2 + (5 \tan x)^2 - 2(5 \tan x).(5 \tan x)\cos(180^\circ - 2x)$ $QT^2 = 50 \tan^2 x - 50 \tan^2 x(-\cos 2x)$ $QT^2 = 50 \tan^2 x(1 + \cos 2x)$ $QT^2 = 50 \tan^2 x(1 + 2 \cos^2 x - 1)$ $QT^2 = 50 \tan^2 x(2 \cos^2 x)$ $QT^2 = 100 \frac{\sin^2 x}{\cos^2 x} (\cos^2 x)$ $QT^2 = 100 \sin^2 x$ $QT = 10 \sin x$ <p>OR/OF</p> $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5 \tan x)^2 = (5 \tan x)^2 + TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ[TQ - 10 \tan x.\cos x]$ $TQ = 10 \tan x.\cos x \quad (TQ \neq 0)$ $= 10 \frac{\sin x}{\cos x}.\cos x$ $= 10 \sin x$	<p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$</p> <p>✓ $\cos 2x = 2 \cos^2 x - 1$ & reduction</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $QT^2 = 100 \sin^2 x$</p> <p style="text-align: right;">(5)</p> <p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$ ✓ quadratic equation into TQ</p> <p>✓ $TQ = 10 \tan x . \cos x$ ✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p style="text-align: right;">(5)</p>
8.3	<p>Area of $\Delta TQR = \frac{1}{2}.TQ.QR \sin \hat{T}QR$</p> $= \frac{1}{2}(10 \sin 25^\circ)(5)(\sin 70^\circ)$ $= 9,93 \text{ unit}^2$	<p>✓ correct substitution into the area rule ✓ answer</p> <p style="text-align: right;">(2)</p>
[10]		

QUESTION/VRAAG 9



9.1	tangents from same(common) point/raaklyne vanaf dieselfde punt	✓ R	(1)
9.2.1	$\hat{S}_2 = \hat{S}_1$ $\therefore \hat{S}_2 = 51^\circ$	[\angle s opp equal sides/ \angle e teenoor gelyke sye] [sum of \angle s in Δ /som van \angle e in Δ]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93^\circ$ $\hat{S}_3 = 42^\circ$	[ext \angle of cyclic quad/buite \angle van koordevh]	✓ R ✓ answer (2)
	OR/OF $\hat{S}_1 = 87^\circ$ $\hat{S}_3 = 180^\circ - (87^\circ + 51^\circ)$ $\hat{S}_3 = 42^\circ$	[opp \angle s of cyclic quad/teenoorst \angle e v kdvh] [\angle s on a str line/ \angle e op reguitlyn]	✓ R ✓ answer (2)
			[5]

QUESTION/VRAAG 10

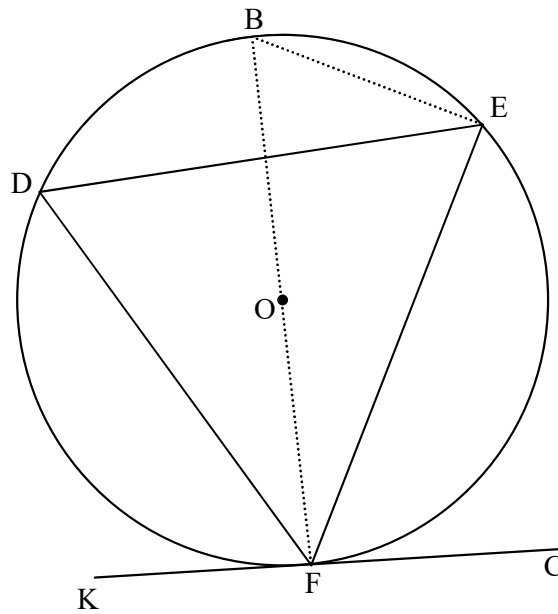


10.1	line from centre \perp to chord/lyn vanaf middelpunt \perp op koord	\checkmark R	(1)
10.2	$\therefore \hat{A}_1 = 90^\circ - x$ [sum of \angle s in Δ /som van \angle e in Δ] $\therefore \hat{M}_1 = 180^\circ - 2x$ [\angle at centre= $2 \times$ at circumf/midpts $\angle = 2 \times$ omtreks \angle]	\checkmark S \checkmark S \checkmark R	(3)
10.3	$\hat{C}\hat{A}\hat{D} = 90^\circ$ [\angle in semi circle/ \angle in halfsirkel] $\hat{A}_2 = 90^\circ - (90^\circ - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.]	\checkmark S \checkmark R \checkmark S \checkmark R	(4)
	OR/OF $\hat{E}\hat{M}\hat{D} = 2x$ [adj suppl \angle s/aanligg suppl \angle e] $\therefore \hat{A}_2 = x$ [\angle at centre= $2 \times$ \angle at circumf/midpts $\angle = 2 \times$ omtreks \angle] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.]	\checkmark S \checkmark S \checkmark R \checkmark R	(4)
	OR/OF $\hat{M}_3 = 180^\circ - 2x$ [vert. opp/ regoorstaande \angle e] $\therefore \hat{A}_3 = 90^\circ - x$ [\angle at centre= $2 \times$ \angle at circumf/midpts $\angle = 2 \times$ omtreks \angle] $\hat{B}\hat{A}\hat{E} = 90^\circ$ [\angle in semi-circle/ \angle in halfsirkel] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.]	\checkmark S \checkmark R \checkmark S \checkmark R	(4)
	OR/OF		

	<p> $CD \parallel AB$ [midpt. Thm/ <i>middelpuntst.</i>] $\hat{B}AE = 90^\circ$ [\angle in semi-circle/\angle in <i>halfsirkel</i>] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt. \angles; $CD \parallel AB$/<i>verwiss \anglee</i>] $\therefore \hat{A}_2 = x = C$ $\therefore AD$ is a tangent [converse tan-chord theorem/<i>omgek rkl-kd st.</i>] </p> <p>OR/OF</p> <p> $\hat{C}AD = 90^\circ$ [\angle in semi circle/\angle in <i>halfsirkel</i>] $AC = \text{diameter}$ [converse \angle in semi circle/<i>omgek \angle in halfsirkel</i>] $\therefore AD$ is a tangent [converse radius \perp tangent/<i>omgek radius \perprkl</i>] </p>	<p> \checkmark S \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R </p> <p>(4)</p> <p>(4)</p>
10.4	<p> $AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore EB = 36$ units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] $AE = 12\sqrt{5}$ or 26,83 units </p> <p>OR/OF</p> <p> $AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] $FE = 6\sqrt{5}$ $AE = 12\sqrt{5}$ or 26,83 units </p>	<p> \checkmark FM = 12 \checkmark R \checkmark EB = 36 \checkmark using Pyth correctly \checkmark answer \checkmark FM = 12 \checkmark R \checkmark EM = 18 \checkmark using Pyth correctly \checkmark answer </p> <p>(5)</p> <p>(5)</p>
		[13]

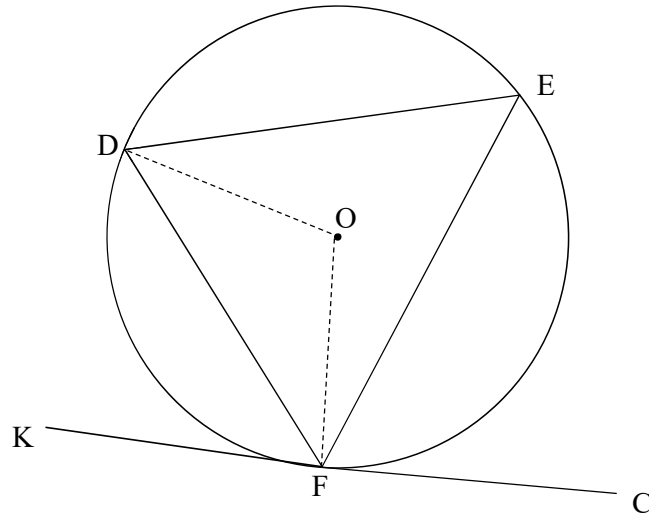
QUESTION/VRAAG 11

11.1



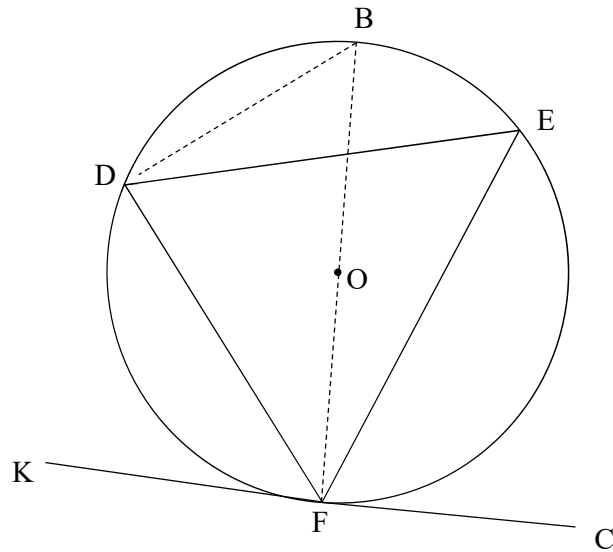
	<p>Construction: Draw diameter BF and draw BE <i>Konstruksie: Trek middellyn BF en verbind BE</i></p> <p>$\hat{B}FK = 90^\circ$ or $\hat{D}FK = 90^\circ - \hat{B}FD$ [radius \perp tangent/<i>raaklyn</i>]</p> <p>$\hat{B}EF = 90^\circ$ [\angle in semi-circle/<i>semi-sirkel</i>]</p> <p>$\therefore \hat{D}EF = 90^\circ - \hat{B}ED$</p> <p>$= 90^\circ - \hat{B}FD$ [\angles same segment/<i>\angle e dieselfde segment</i>]</p> <p>$\therefore \hat{D}FK = \hat{D}EF$</p>	<p>✓ Constr</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
--	--	---

OR/OF



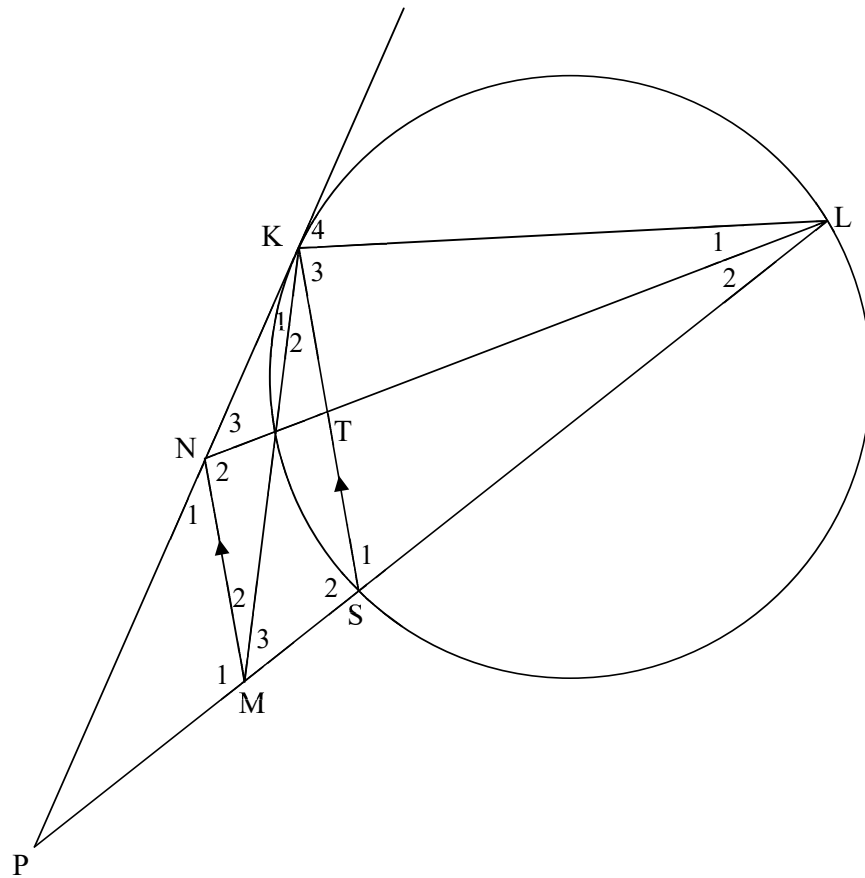
	<p>Construction: Draw radii DO and OF</p> <p><i>Konstruksie: Trek radii DO en OF</i></p> <p>$\hat{O}F\hat{K} = 90^\circ$ or $\hat{D}F\hat{K} = 90^\circ - \hat{O}F\hat{D}$ radius \perp tangent/raaklyn]</p> <p>$\hat{O}D\hat{F} = \hat{O}F\hat{D}$ [\angles opp = sides/\anglee teenoor = sye]</p> <p>$\therefore \hat{D}O\hat{F} = 180^\circ - 2\hat{O}F\hat{D}$ [\angles of Δ/\anglee van Δ]</p> <p>$\hat{D}E\hat{F} = 90^\circ - \hat{O}F\hat{D}$ [\angle at centre = $2 \times \angle$ circumf/ midpts $\angle = 2 \times$ omtreks \angle]</p> <p>$\therefore \hat{D}F\hat{K} = \hat{D}E\hat{F}$</p>	<p>✓ construction</p> <p>✓ S ✓R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
--	--	--

OR/OF



	<p>Construction: Draw diameter BF and join BD.</p> <p><i>Konstruksie: Trek middellyn BF en verbind BD.</i></p> <p>$\hat{B}\hat{F}K = 90^\circ$ or $\hat{D}\hat{F}K = 90^\circ - \hat{B}\hat{F}D$ [radius \perp tangent/raaklyn]</p> <p>$\hat{F}\hat{D}B = 90^\circ$ [\angle in half circle/semi-sirkel]</p> <p>$\hat{B} = 90^\circ - \hat{B}\hat{F}D$</p> <p>$\therefore \hat{D}\hat{F}K = \hat{B}$</p> <p>but $\hat{B} = \hat{E}$ [\angles same segment/\anglee dieselfde segment]</p> <p>$\therefore \hat{D}\hat{F}K = \hat{E}$</p>	<p>✓ construction</p> <p>✓ S ✓/R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
--	--	---

11.2



<p>11.2.1(a)</p>	<p>$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/raaklynkoordstelling] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp \angles; / ooreenk \angles; MN KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = \hat{NML}$</p>	<p>✓ S ✓ R ✓ S ✓ R</p> <p>(4)</p>
<p>11.2.1(b)</p>	<p>$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = \hat{NML}$ \therefore KLMN is a cyclic quad [ext \angle of quad = opp int \angle / <i>buite \angle van vh = teenoorst binne \angle</i>] OR/OF $N_1 = \hat{K}_1 + \hat{K}_2 = \hat{NKS}$ [corresp \angles; / ooreenk \angles; MN KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / raaklynkoordstelling] $\hat{N}_1 = \hat{KLS}$ \therefore KLMN is a cyclic quad [ext \angle of quad = opp int \angle / <i>buite \angle van vh = teenoorst binne \angle</i>] OR/OF</p>	<p>✓ R</p> <p>(1)</p> <p>✓ R</p> <p>(1)</p>

	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore KLMN$ is a cyclic quad [opp. \angle s supplementary]	\checkmark R (1)
11.2.2	<p>In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{L}_1 = \hat{M}_2$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $= \hat{K}_2$ [alt \angle s; / verw \angle e; $MN \parallel KS$] $N\hat{K}L = M\hat{S}K$ [\angle s of Δ / \angle e van Δ] $\triangle LKN \parallel \triangle KSM$ <p>OR/OF In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $N\hat{K}L = \hat{M}_1$ [ext \angle of cyclic quad/buite \angle van koordevh] $= \hat{S}_2$ [corresp \angle s/ooreenk \angle e; $KS \parallel NM$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle] <p>OR/OF In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [\angle s on straight line / \angle e op reguitlyn] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle]	\checkmark S \checkmark R \checkmark S \checkmark S/R \checkmark S \checkmark S \checkmark R \checkmark S/R \checkmark S \checkmark R \checkmark S \checkmark R \checkmark S/R \checkmark S \checkmark R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\triangle LKN \parallel \triangle KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	\checkmark S \checkmark R \checkmark substitution \checkmark answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line \parallel one side of Δ / lyn \parallel een sy v Δ] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10,95$	\checkmark SM = 6 \checkmark S \checkmark R \checkmark answer (4)
		[23]

TOTAL/TOTAAL: 150