



**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

NOVEMBER 2022

MATHEMATICS P2

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, including an information sheet and an answer book of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The following table shows a sleeping pattern record, in hours, of ten Grade 11 learners:

Learner	1	2	3	4	5	6	7	8	9	10
Number of hours slept	7	8	8	5	6	3	4	8	7	10

- 1.1 Calculate the mean number of hours slept by the learners. Give the answer correct to TWO decimal places. (1)
- 1.2 Write down the five-number summary for this data. (2)
- 1.3 Draw a box-and-whisker diagram for this data set. (2)
- 1.4 Refer to your diagram and comment on the skewness of the data, and give a reason for your answer. (2)
- 1.5 Calculate the standard deviation for this data. Give your answer correct to TWO decimal places. (2)
- 1.6 A learner is considered to have slept well, if his sleeping time is above one standard deviation from the mean. How many learners slept well? (2)

[11]

QUESTION 2

The different ages of teachers at a certain school in the Eastern Cape are given in the table below.

2.1 Complete the following table in your ANSWER BOOK.

AGE	FREQUENCY	CUMULATIVE FREQUENCY
$25 < A \leq 30$	2	
$30 < A \leq 35$	8	
$35 < A \leq 40$	4	
$40 < A \leq 45$	5	
$45 < A \leq 50$	11	
$50 < A \leq 55$	19	
$55 < A \leq 60$	20	
$60 < A \leq 65$	6	

(2)

2.2 Draw an ogive on the set of axes provided in your ANSWER BOOK to represent the data in the table.

(4)

2.3 Use your graph to find an estimate of the median age.

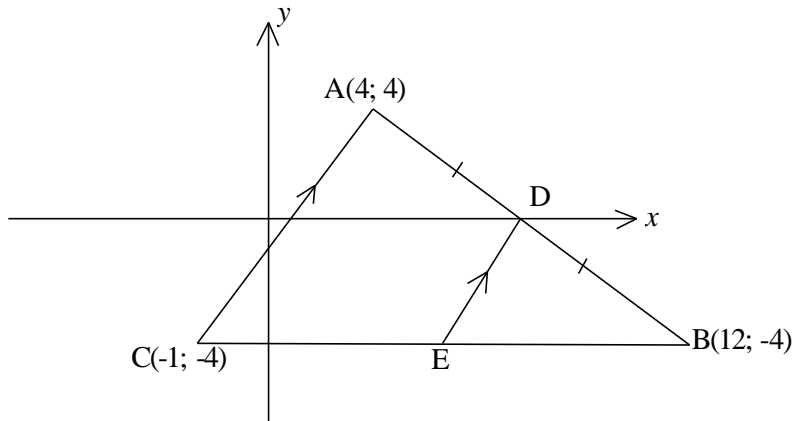
(2)

2.4 The school would like to give all teachers older than 57 a special present. Use your graph to find an estimate for the percentage of teachers older than 57 years of age.

(2)
[10]

QUESTION 3

In the diagram below, the coordinates of $A(4; 4)$, $B(12; -4)$ and $C(-1; -4)$ are given. $AC \parallel DE$ and CEB is a straight line. D is the midpoint of AB .

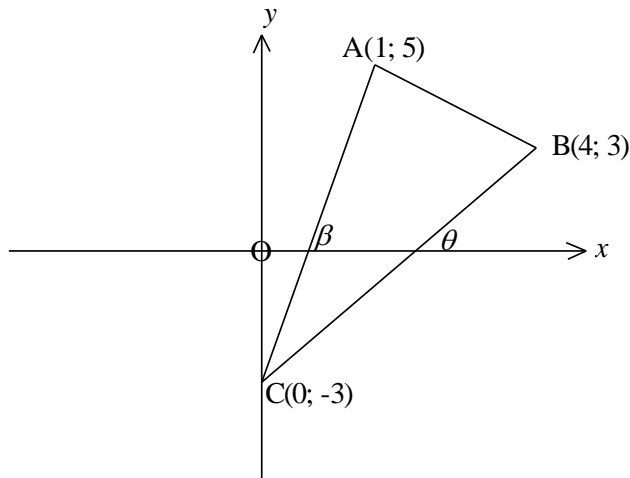


Determine:

- 3.1 The length of AB . Give your answer correct to TWO decimal places (2)
 - 3.2 The coordinates of D , the midpoint of AB (2)
 - 3.3 The equation of line DE (4)
 - 3.4 The coordinates of E (3)
- [11]**

QUESTION 4

A(1; 5), B(4; 3) and C(0; -3) are vertices of the triangle given below.



4.1 Determine, using any method, the coordinates of D if ABCD is a parallelogram. (2)

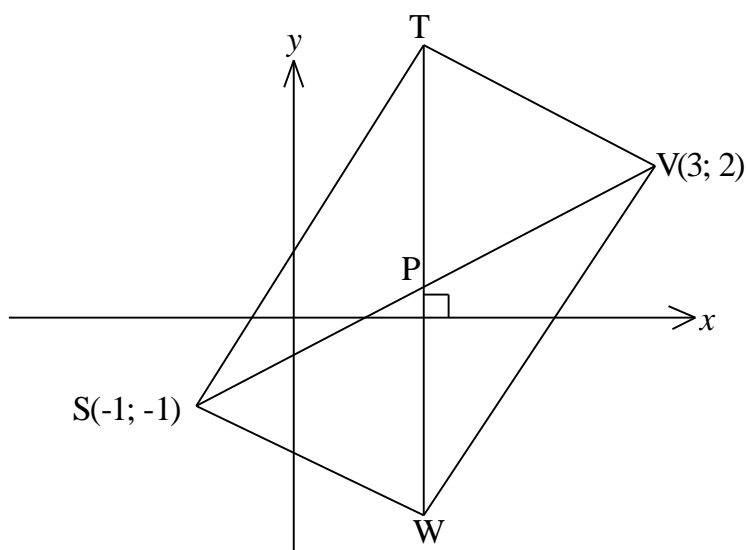
4.2 If the distance between C and F(8; p) is 12 units, determine the value(s) of p (to the nearest integer). (5)

4.3 Determine the size of \widehat{ACB} . (5)

[12]

QUESTION 5

In the diagram below, the diagonals of STVW are equal in length and bisect each other at P. Calculate the coordinates of T and W.



(6)
[6]

QUESTION 6

6.1 If $-3 \sin \beta - 2 = 0$ and $\beta \in [0^\circ; 270^\circ]$, use a sketch in the correct quadrant to determine the value of: $1 + \tan^2 \beta$ without a calculator. (5)

6.2 If, $\cos 75^\circ = m$ express each of the following in terms of m , showing all your working:

6.2.1 $\cos^2 105^\circ$ (2)

6.2.2 $\sin 15^\circ$ (2)

6.2.3 $\tan 15^\circ$ (2)

6.3 Given the expression:

$$\frac{\cos(180^\circ - k) \cdot \sin(k - 90^\circ) - 1}{\tan^2(540^\circ + k) \cdot \sin(90^\circ + k) \cdot \cos(-k)}$$

6.3.1 Simplify the expression. (7)

6.3.2 Determine the values of $k \in [0^\circ; 360^\circ]$ for which the expression is undefined. (6)

6.4 Prove that:

$$\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = \frac{4 \tan \theta}{\cos \theta} \quad (5)$$

6.5 Determine the general solution of:

$$6 \sin^2 \theta + \cos \theta = 4 \quad (7)$$

6.6 If $p = \tan A + \sin A$ and $q = \tan A - \sin A$, prove that:

$$pq = \tan^2 A \cdot \sin^2 A \quad (5)$$

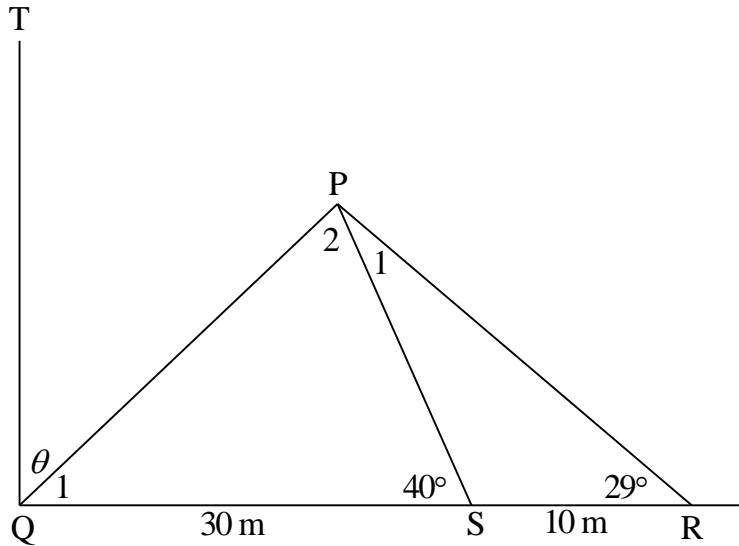
[41]

QUESTION 7

Study the diagram below and then answer the questions that follow.

$\widehat{TQR} = 90^\circ$, $\widehat{QSP} = 40^\circ$, $\widehat{SRP} = 29^\circ$, $QS = 30$ m and $SR = 10$ m.

It is also given that $\widehat{TQP} = \theta$.

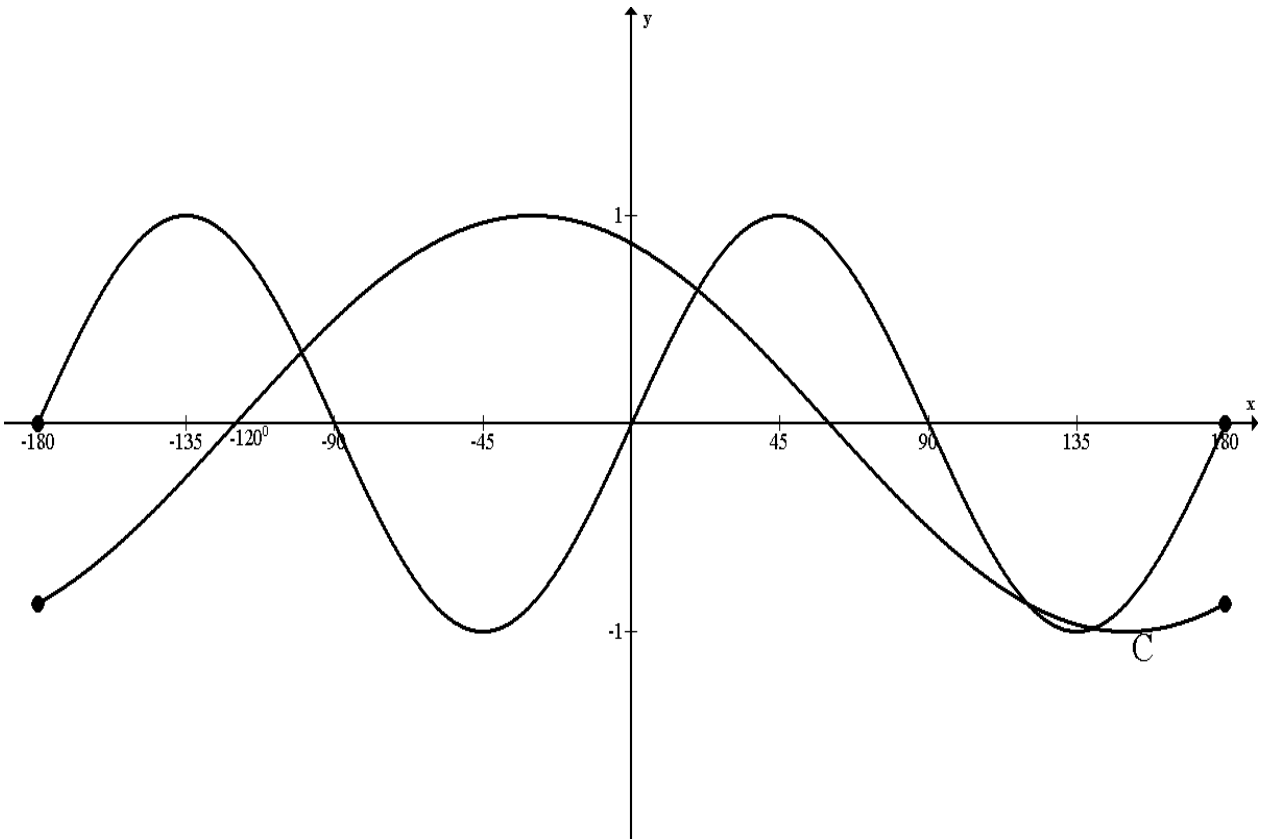


- 7.1 Give a reason why $\widehat{P}_1 = 11^\circ$. (1)
- 7.2 Calculate the length of PS. (3)
- 7.3 Determine the value of θ , correct to the nearest degree. (5)

[9]

QUESTION 8

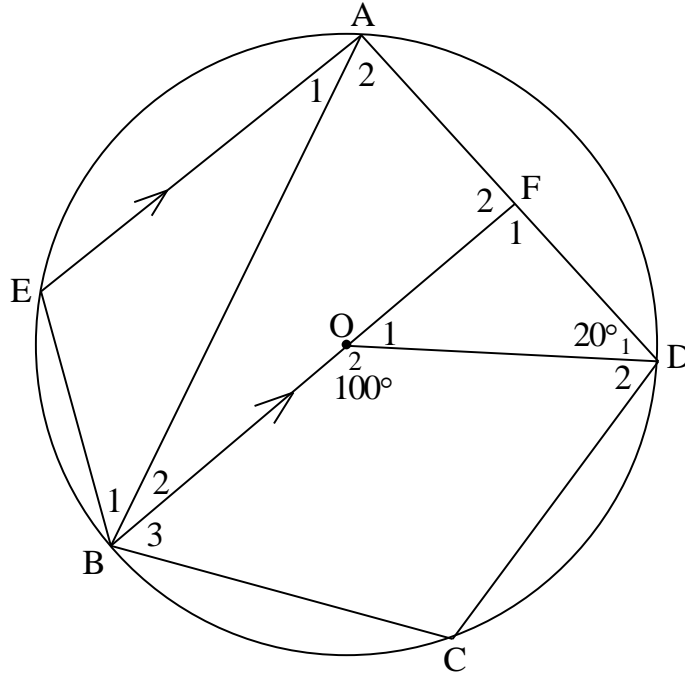
The sketch graphs of $f(x) = \sin ax$ and $g(x) = \cos(x - b)$ is given below.



- 8.1 Determine the values of a and b . (2)
 - 8.2 Determine the coordinates of C, a turning point on $g(x)$. (2)
 - 8.3 For which values of x , where $x < 0$, is $f(x) \cdot g(x) \geq 0$? (3)
 - 8.4 Determine the equation of $f(x)$ if the y -axis is moved 30° to the left. (1)
- [8]**

QUESTION 9

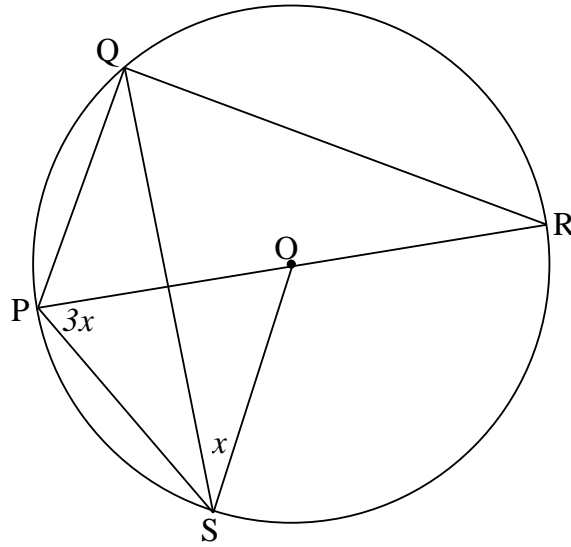
9.1 In the diagram below, O is the centre of circle AEB CD, with line BOF \parallel EA. F lies on AD, $\widehat{BOD} = 100^\circ$ and $\widehat{D}_1 = 20^\circ$. The sizes of some of the angles are given in the table below. In each case, supply a valid reason.



	STATEMENT	REASONS
9.1.1	$\widehat{A}_2 = 50^\circ$	
9.1.2	$\widehat{O}_1 = 80^\circ$	
9.1.3	$\widehat{F}_1 = 80^\circ$	
9.1.4	$\widehat{A}_1 = 30^\circ$	
9.1.5	$\widehat{B}_2 = 30^\circ$	

(5)

9.2 P, Q and R are points on the circumference of the circle with centre O. PR is the diameter of the circle. $\widehat{QSO} = x$ and $\widehat{OPS} = 3x$.



Express each of the following in terms of x , giving a reason for your answer:

9.2.1 \widehat{SQR} (2)

9.2.2 \widehat{PQS} (3)

9.2.3 \widehat{PSQ} (3)

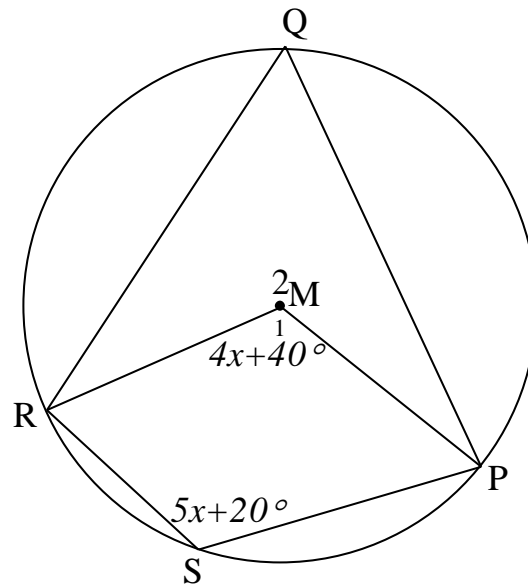
9.2.4 \widehat{PRQ} (2)

9.2.5 \widehat{QPR} (2)

[17]

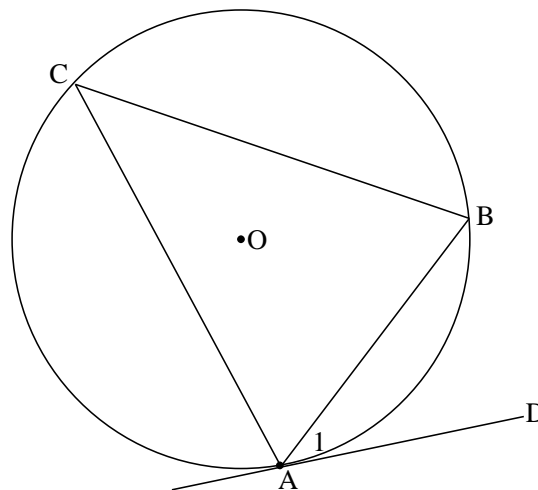
QUESTION 10

- 10.1 In the figure, P, Q, R and S are points on the circumference of a circle with centre M. It is given that $\widehat{M}_1 = 4x + 40^\circ$ and $\widehat{S} = 5x + 20^\circ$.



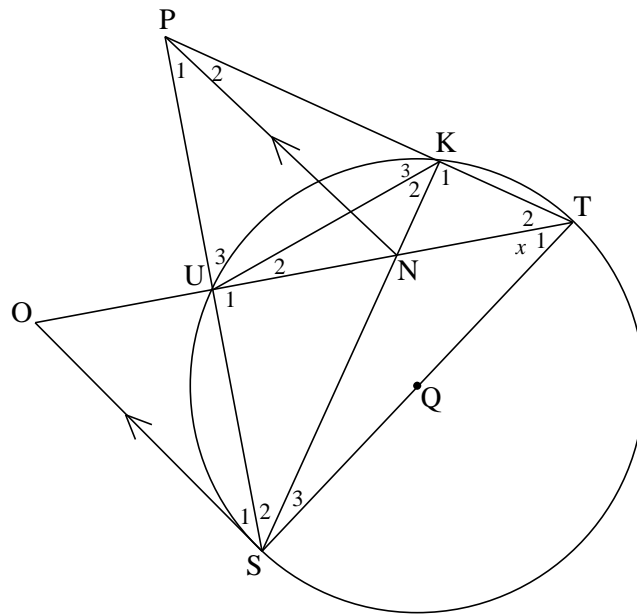
Calculate the size of \widehat{Q} with reasons. (5)

- 10.2 In the diagram below, the circle with centre O passes through the points A, B and C. AD is a tangent to the circle at A.



Use the diagram to prove the theorem that states that $\widehat{A}_1 = \widehat{C}$. (6)

10.3 Refer to the diagram below. ST is a diameter of the circle. OS \parallel PN, TO bisects \widehat{STP} . Let $\widehat{T}_1 = x$.



10.3.1 Prove that PUNK is a cyclic quadrilateral. (5)

10.3.2 SO is a tangent to circle KUST. (6)

10.3.3 Prove that POST is a cyclic quadrilateral. (3)

[25]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$