basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

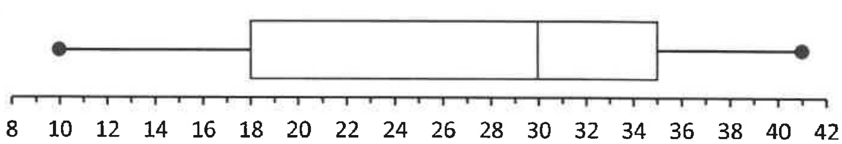
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

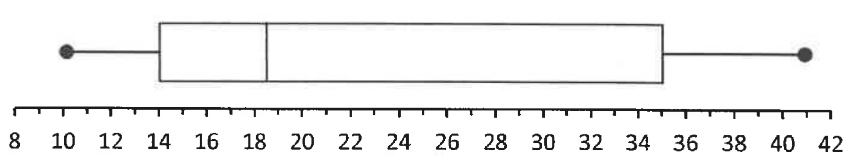
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)

[10]

QUESTION 2

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)

2.2 Determine the equation of the least squares regression line for the data. (3)

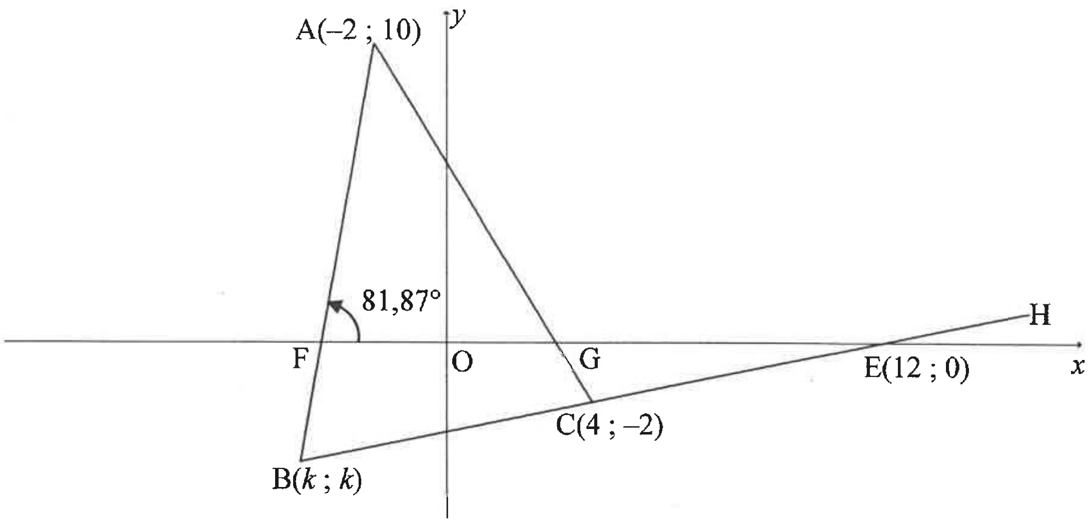
2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)

2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)

[10]

QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

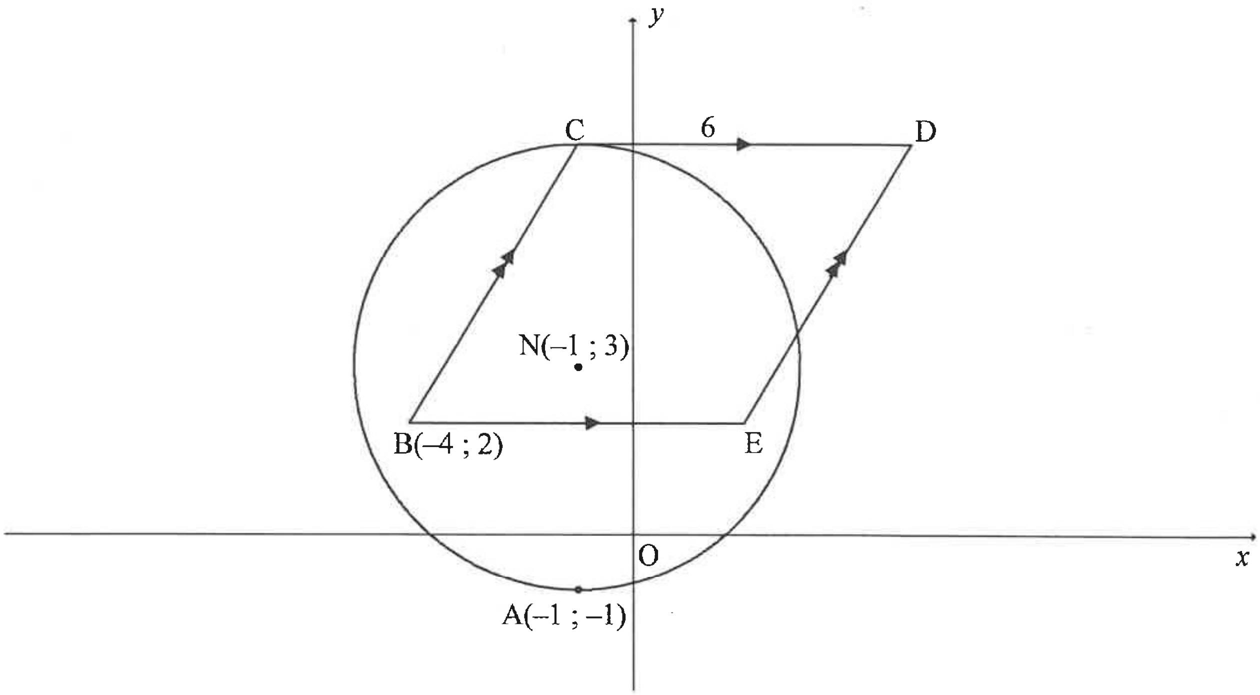
(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

(b) Tangent to the circle at point B(k ; k) (3)

[24]

QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

Calculate the:

4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)

[15]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$$ (6)

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)
[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

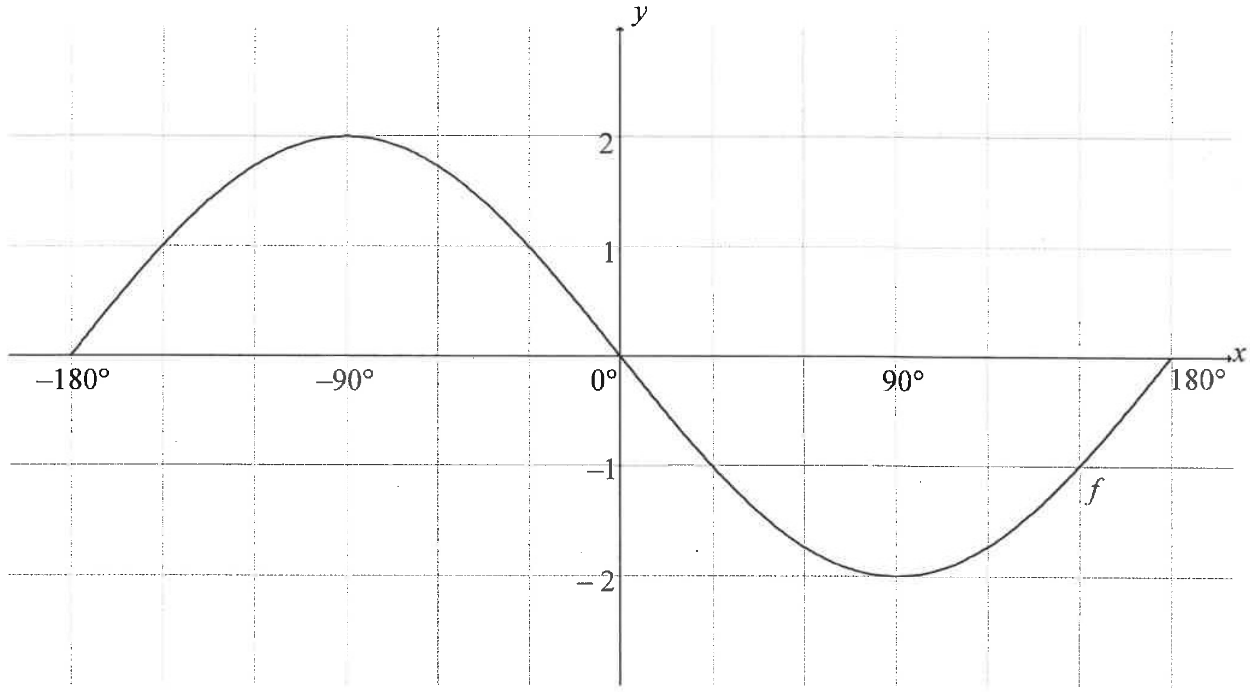
6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

6.1.2 Simplify completely: $2\cos 6x\cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)
[15]

QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180°\,;\,180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180°\,;\,180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

7.2 Write down the period of $f(3x)$. (2)

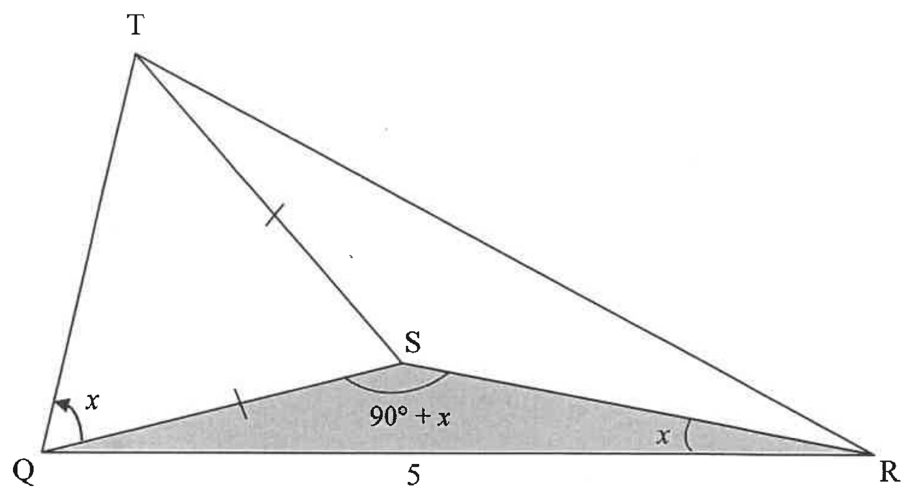
7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180°\,;\,180°]$ for which $f(x) - g(x) = 1$. (1)

7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)

[8]

QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.
$Q\hat{S}R = 90° + x$, $Q\hat{R}S = x$, $QR = 5$ units and $TS = SQ$.



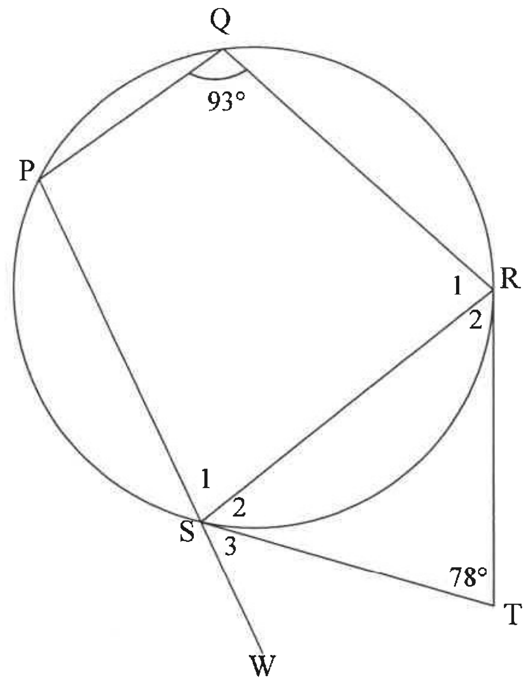
8.1 Prove that $QS = 5 \tan x$ (3)

8.2 Prove that the length of $QT = 10 \sin x$ (5)

8.3 Calculate the area of ΔTQR if $T\hat{Q}R = 70°$ and $x = 25°$. (2)
[10]

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T}=78°$ and $\hat{Q}=93°$.



9.1 Give a reason why ST = TR. (1)

9.2 Calculate, giving reasons, the size of:

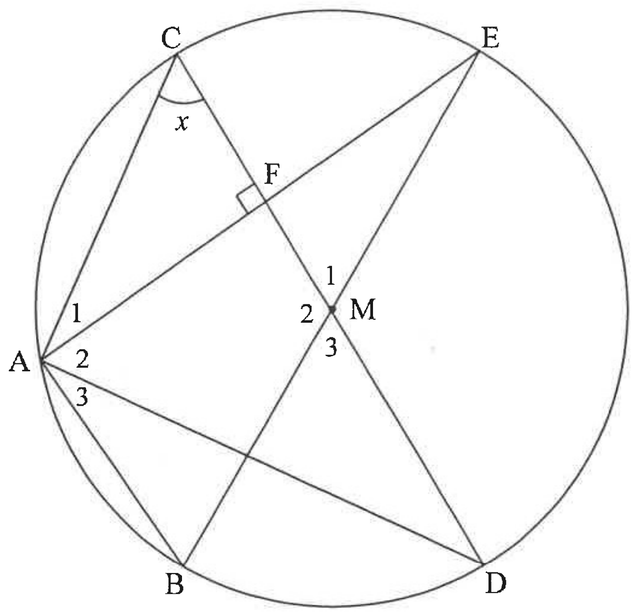
9.2.1 $\hat{S}_2$ (2)

9.2.2 $\hat{S}_3$ (2)

[5]

QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. $AE \perp CD$. Let $\hat{C} = x$.



10.1 Give a reason why AF = FE. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

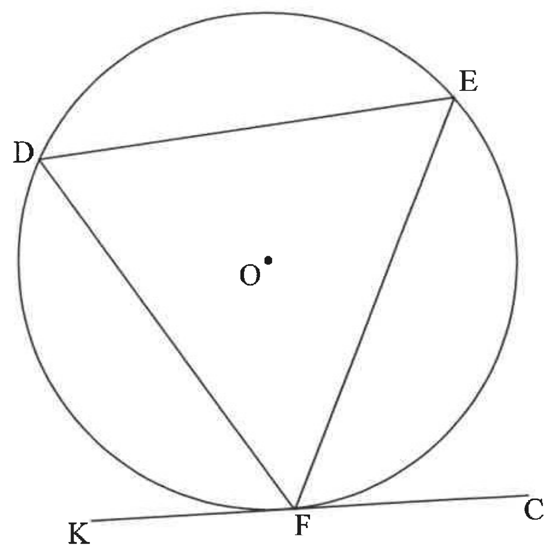
10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)

[13]

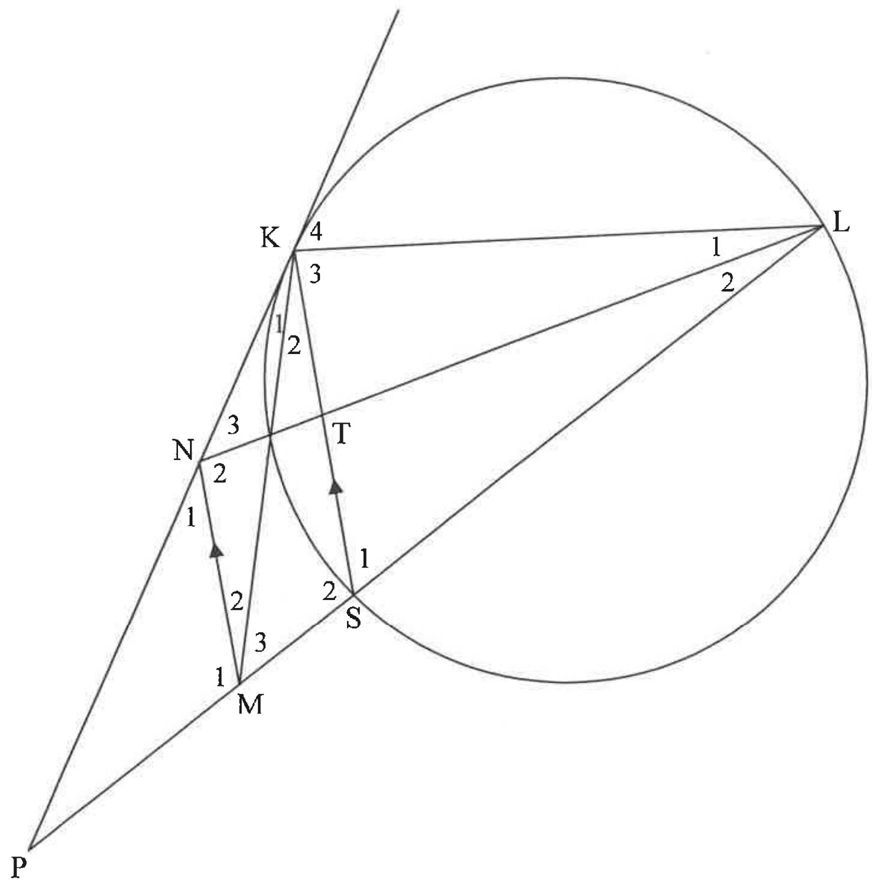
QUESTION 11

11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN || SK. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\Delta LKN \,|||\, \Delta KSM$. (5)

11.2.3 If $LK = 12$ units and $3KN = 4SM$, determine the length of KS. (4)

11.2.4 If it is further given that $NL = 16$ units, $LS = 13$ units and $KN = 8$ units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$