NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) Writing equation in standard form

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-3)}}{2(3)}$$

$x = 2{,}14$ or $x = -0{,}47$

(2) $x < -0{,}47$ or $x > 2{,}14$

(b) $y = 12x$

$12x = x^2 + 5x$
$0 = x^2 - 7x$
$x = 0$ or $x = 7$

$y = 0$ or $y = 84$

(c) $\sqrt{x+7} = x - 5$
$x + 7 = x^2 - 10x + 25$
$0 = x^2 - 11x + 18$
$0 = (x-9)(x-2)$
$x = 9$ or $x = 2$ N/A

(d) $177\,146 = \frac{2(3^n - 1)}{(3-1)}$

$177\,147 = 3^n$

$n = 11$

QUESTION 2

(a) $\lim\limits_{h \to 0} \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h}$
$\lim\limits_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$
$\lim\limits_{h \to 0} \frac{2xh + h^2 - 5h}{h}$
$\lim\limits_{h \to 0} \frac{h(2x + h - 5)}{h}$
$\lim\limits_{h \to 0} (2x + h - 5)$
$f'(x) = 2x - 5$

(b) $g(x) = x^{\frac{1}{3}} + 6x^{-1}$

$g'(x) = \frac{1}{3}x^{-\frac{2}{3}} - 6x^{-2}$

(c) $f'(x) = -2x + 3$

$-2x + 3 = -1$

$x = 2$

$(2;6)$

$6 = -2 + p$

$p = 8$

QUESTION 3

(a) (1) $x \in (-\infty;\infty)$ or $x \in \mathbb{R}$

(2) $y \in (-2;\infty)$ or $\{y : y > -2\}$

(3) $y = -3^x + 2$

(4) $3^x - 2 = 0$

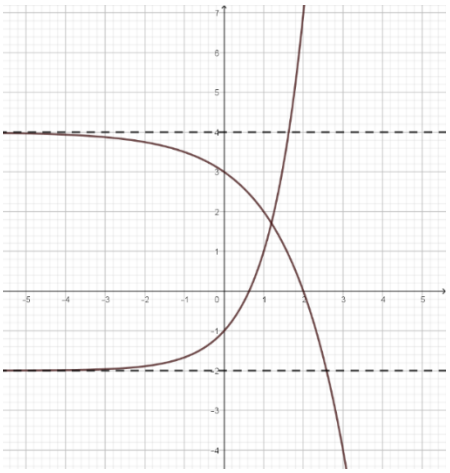
$3^x = 2$

$x = \log_3 2$

$x = 0,63$

$x \geq 0,63$ or x ≥ 0,6

(b) (1)



y-intercept

x-intercept

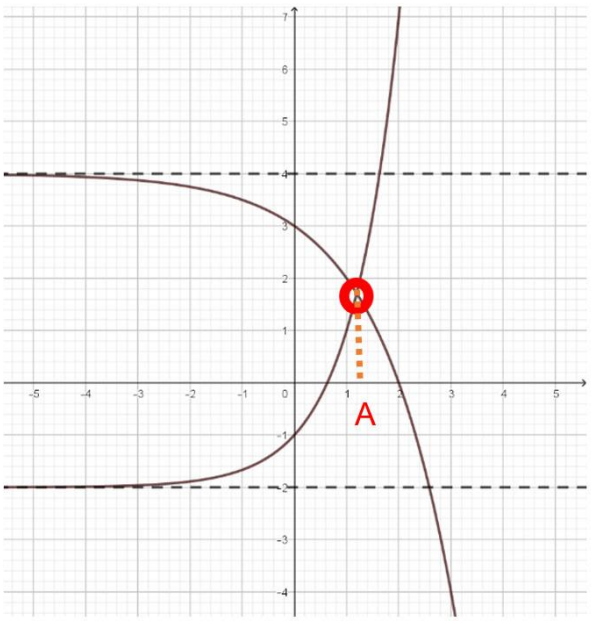
Shape (Take note of the asymptote)

(2) $y = -2^x + 7$

(3) $3^x - 2 = -2^x + 4$

$3^x + 2^x = 6$

Shown at A



QUESTION 4

(a) $1 = (2) + t$

$t = -1$

(b) $\log_3 x = 1$

$C(3;1)$ for the x-value of 3

(c) $0 = \dfrac{5}{x-2} + 1$

$-1(x-2) = 5$

$x = -3$

$0 = \log_3 x$

$3^0 = x$

$x = 1$

AB = 4 units

(d) $x = \log_3 y$

$y = 3^x$

(e) $x \in (1;2)$

QUESTION 5

(a) Method mark for workings

$2a = 4$

$a = 2$

$3(2) + b = 5$

$b = -1$

$2 - 1 + c = 4$

$c = 3$

$T_n = 2n^2 - n + 3$

(b) $2n^2 - n + 3 = 949$

$2n^2 - n - 946 = 0$

$n = 22$ or $n \neq -\frac{43}{2}$

(c) $\sum_{n=1}^{22}\left(2n^2 - n + 3\right)$

QUESTION 6

(a) $F_v = \dfrac{5000\left[\left(1 + \dfrac{0{,}15}{12}\right)^{36} - 1\right]}{\dfrac{0{,}15}{12}}$ Fv formula interest number of payments

$F_v = \text{R}225\,577{,}53$

(b) $2\,500\,000 = \dfrac{5\,000\left[\left(1 + \dfrac{0{,}15}{12}\right)^{n} - 1\right]}{\dfrac{0{,}15}{12}}$ $\left(= 2\ 500\ 000\right)$ I and n Fv Formula

$n = 160$ months workings number of months

SECTION B

QUESTION 7

(a) $850\ 000 = \dfrac{x\left[1-\left(1+\dfrac{0{,}09}{12}\right)^{-240}\right]}{\dfrac{0{,}09}{12}}$ Present value formula i and n subs

$x = \text{R}7\,647{,}67$

(b) $850\,000\left(1+\dfrac{0.09}{12}\right)^{144} - \dfrac{9\,000\left[\left(1+\dfrac{0.09}{12}\right)^{144}-1\right]}{\dfrac{0.09}{12}}$

Compound growth
minus Fv Formula
Interest rate
144

Balance Outstanding $= \text{R}173\,507{,}13$

QUESTION 8

(a) $\log_3 x - \log_3 (x-5) = 1$

$$\log_3 \frac{x}{x-5} = 1$$

$$\frac{x}{x-5} = 3$$

$$x = \frac{15}{2} \text{ or } x = 7{,}5$$

(b) $ar = -24$
$ar^2 + ar^3 = -18$

$$a = \frac{-24}{r}$$

$$-24r - 24r^2 = -18$$

$$0 = 4r^2 + 4r - 3$$

$$0 = (2r+3)(2r-1)$$

$$r = -\frac{3}{2} \text{ or } r = \frac{1}{2}$$

Series is converging therefore $r = \frac{1}{2}$

(c) (1) $58\,000 = 25\,000 + (12-1)d$
$33\,000 = 11d$
$d = 3\,000$

(2) $S_n = \frac{12}{2}\left(2(25\,000) + (12-1)(3\,000)\right)$

$S_n = 498\,000$

Total income from ticket sales

R25$\times$498 000

$=$ R12 450 000

QUESTION 9

(a) $N = 8$

$M = 6$

$y = ax(x-6)$

$2 = a(4)(4-6)$

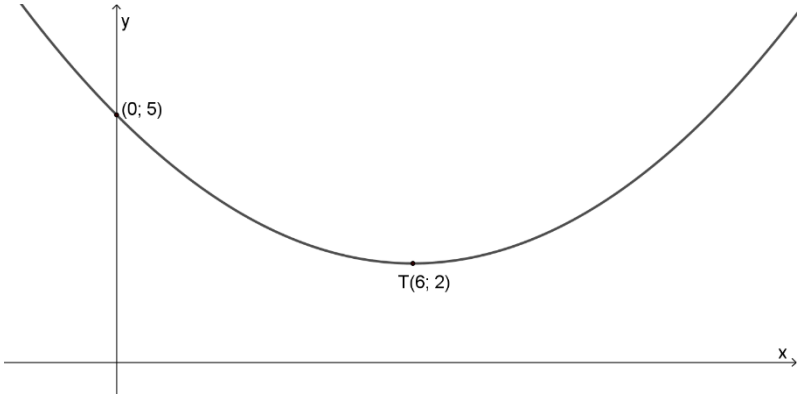
$a = -\frac{1}{4}$

$y = -\frac{1}{4}(3)(3-6)$ sub in x-value of 3

$y = \frac{9}{4}$

$H(3;2,25)$

(b)



Any y-intercept above the turning point
Turning point x value
Turning point y value
Shape

QUESTION 10

(a) $x^3 - 5x^2 + 3x + 9 = 3x + 9$

$x^2(x-5) = 0$

$x = 0$ or $x = 5$

Coordinates of A

$y = 3(5) + 9 = 24$

$A(5;24)$

$f'(x) = 3x^2 - 10x + 3$

$f'(5) = 3(5)^2 - 10(5) + 3 = 28$

$y = 28x + c$

$24 = 28(5) + c$

$c = -116$

$0 = 28x - 116$

$x = \frac{29}{7}$

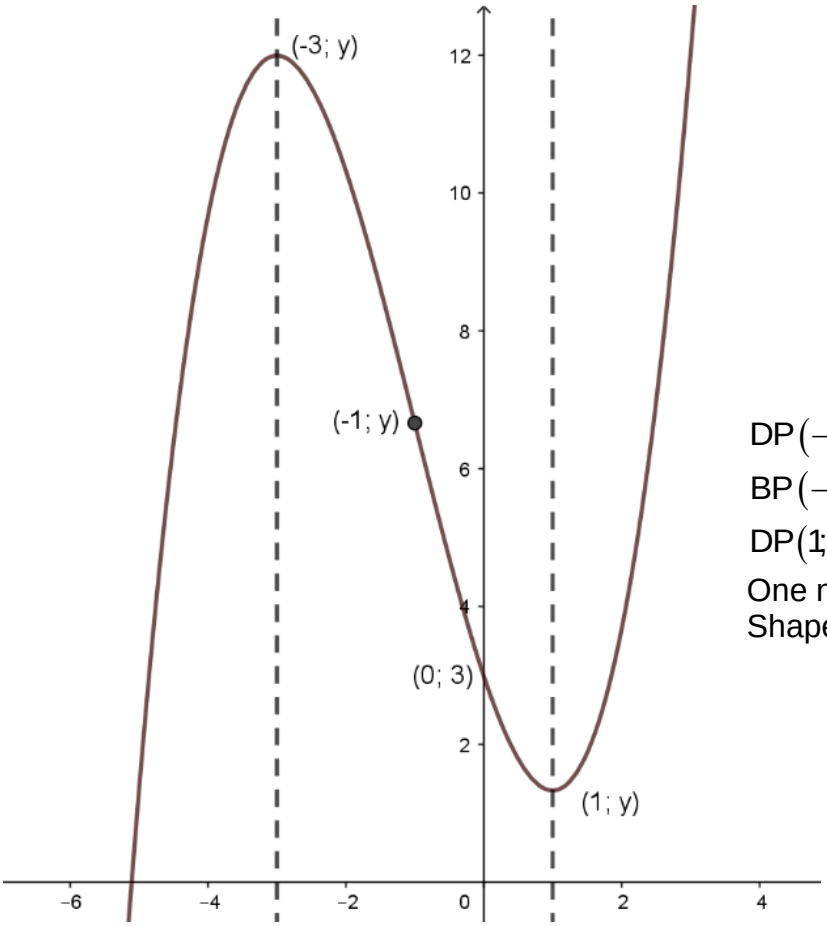
$B\left(\frac{29}{7};0\right)$

(b) (1) $0=(x+1)^2-4$

$0=x^2+2x-3$
$0=(x+3)(x-1)$
$x=-3$ or $x=1$

$f(x)$ is increasing when
$x\in(-\infty;-3]\cup[1;\infty)$

(2)



DP$(-3;\ y)$
BP$(-1;\ y)$
DP$(1;\ y)$
One negative intercept
Shape

QUESTION 11

(a) $x(x+y)=1000$ or $x^2+xy=1000$

$$y=\frac{1000}{x}-x$$

(b) $F=5x+2y$

$$F=5x+2\left(\frac{1000}{x}-x\right)$$

$$F=5x+\frac{2\,000}{x}-2x$$

$$F=3x+\frac{2\,000}{x}$$

$$\frac{dF}{dx}=3-\frac{2\,000}{x^2}$$

$$3-\frac{2\,000}{x^2}=0$$

$$x=\sqrt{\frac{2\,000}{3}}$$

or

$x=25{,}82$ units

QUESTION 12

(a) (1) 5! or 120

(2) $3 \times 4!$ or 72

(3) $2 \times 4!$ (Number of codes with letters together)
Probability of codes together
$\frac{48}{120} = 0,4$

Probability that letters will never be next to each other
$1 - 0,4 = 0,6$

(b) For the series to converge on the number 10 then
Options:
First term must be a 2 and the value of r must be 4/5
First term must be a 4 and the value of r must be 3/5
First term must be 5 and the value of r must be $\frac{1}{2}$ or $\frac{2}{4}$ or $\frac{3}{6}$
First term must be 6 and the value of r must be 2/5
$6 \times \left(\frac{1}{6}\right)^3 = \frac{1}{36}$

Total: 150 marks