NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)(1)	Lower Quartile: 5,8 mm Upper Quartile: 6,0 mm IQR = 6,0 – 5,8 = 0,2 mm	Q_1=5,8 mm Q_3=6,0 mm IQR = 0,2 mm
(a)(2)	P_{50}: 50% × 400 = 200th P_{50} = 5,9 mm	5,9 mm
(a)(3)	$\frac{100}{400}\times 100\%$ $= 25\%$ defective	25% defective
(b)(1)	Negatively skewed.	Negatively skewed
(b)(2)	25% lies between Q_1 and the median (2 to 5) and 25% lies between Q_3 and endpoint. Hence statement is false.	False
(b)(3)	$Q_3 + 1{,}5 \times \text{IQR} = 6 + 1{,}5 \times 4 = 12$ The learner is not an outlier.	6 4 12 and not an outlier

QUESTION 2

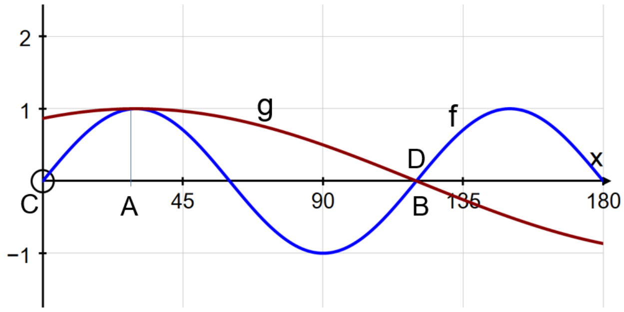
(a)	$\tan\theta = -\frac{1}{3}$ $\theta = 18,4^\circ$	$\tan\theta = -\frac{1}{3}$ $\theta = 18,4^\circ$
(b)	$m_{AB} = -\frac{1}{3}$ $y = -\frac{1}{3}x + c \quad \text{sub.}(-3;10)$ $10 = -\frac{1}{3}\left(-\frac{3}{1}\right) + c$ $c = 9$ $y = -\frac{1}{3}x + 9$ **Alternate:** $m_{AB} = -\frac{1}{3}$ $y - y_1 = m(x - x_1)$ $y - 10 = -\frac{1}{3}(x + 3)$	$m_{AB} = -\frac{1}{3}$ $10 = -\frac{1}{3}\left(-\frac{3}{1}\right) + c$ $y = -\frac{1}{3}x + 9$ $m_{AB} = -\frac{1}{3}$ $y - y_1 = m(x - x_1)$ $y - 10 = -\frac{1}{3}(x + 3)$
(c)	$m_{AD} = 3$ $y = 3x + c \quad \text{sub.} (-3;10)$ $10 = 3(-3) + c$ $c = 19$ $y = 3x + 19$ **Alternate:** $m_{AD} = 3$ $y - y_1 = m(x - x_1)$ $y - 10 = 3(x + 3)$	$m_{AD} = 3$ $10 = 3(-3) + c$ $y = 3x + 19$ $m_{AD} = 3$ $y - y_1 = m(x - x_1)$ $y - 10 = 3(x + 3)$

(d)(1)	For D$(x;y)$: $3x+19=-\frac{1}{3}x-1$ $\frac{10}{3}x=-20$ $x=-6$ $\therefore y=1$ D$(-6;1)$ and A$(-3;10)$ Length AD $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Length AD $=\sqrt{90}=3\sqrt{10}$ $\approx 9{,}5$ units	D$(x;y)$: $3x+19=-\frac{1}{3}x-1$ $x=-6$ $\therefore y=1$ Sub.in: their values $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Length AD $=3\sqrt{10}$
(d)(2)	Eq. of line BC is given as $x=6$ For B$(x;y)$ sub. $x=6$ in $y=-\frac{1}{3}x+9$ $\therefore y=7$ $\therefore$ B$(6;7)$ Using distance formula: length AB $=\sqrt{90}=3\sqrt{10}$ Length AD $=3\sqrt{10}$ … from (d) $\therefore \Delta$ABD is isosceles	sub. $x=6$ in $y=-\frac{1}{3}x+9$ $\therefore y=7$ AB $=3\sqrt{10}$ AD $=3\sqrt{10}$ Conclusion

QUESTION 3

(a)(1)	$x+6=41$ $x=35$	$x+6=41$ $x=35$
(a)(2)	$\{25;38;41;44;48\}$ Using a calculator: $\text{SD}=7,8$	$\{25;38;41;44;48\}$ $\text{SD}=7,8$
(a)(3)	Mean = 39,2 SD Range: $31,4 \le x \le 47$ $\therefore$ 3 scores	Mean = 39,2 $31,4 \le x \le 47$ 3 scores
(b)(1)	Negative	Negative
(b)(2)	$S=-1,8(10)+22,7$ $S=4,7$ Either 4 or 5 calls	$S=-1,8(10)+22,7$ $S=4,7$
(b)(3)	$S=-1,8(3)+22,7$ $S=17,3$ if modelled on the regression equation. However, given that $S=8$ when temp is 3°C Therefore, correlation will still be negative but weaker. **Alternate:** The correlation will increase slightly less (less negative).	$S=17,3$ weaker

QUESTION 4

(a)	Period: $120°$	Period: $120°$
(b)	$y = \cos(x - 30°)$ $y = \cos(180° - 30°)$ $y = -\frac{\sqrt{3}}{2}$ Range = $\left[-\frac{\sqrt{3}}{2} ; 1\right]$	$\left[-\frac{\sqrt{3}}{2} ; 1\right]$
(c)		
(c)	$\sin 3x = \cos(x - 30°)$ at Points A and B	See graph
(d)	$\cos(x - 30°) > \sin 3x$ for: $0° \leq x \leq 120°$	

QUESTION 5

(a)	$\hat{C}_2 = x$ ($\angle$ in same seg.) $\hat{D}_3 = x$ (isosceles Δ)	$\hat{C}_2 = x$ ($\angle$ in same seg.) $\therefore \hat{D}_3 = x$
(b)(1)	$\hat{C}_1 + \hat{D}_2 = 180° - 94°$ (int. $\angle$s of Δ) $\hat{C}_1 = \hat{D}_2$ (CO and OD are radii) (Angles opp. Equal sides) $\therefore \hat{D}_2 = \frac{180° - 94°}{2}$ $\therefore \hat{D}_2 = 43°$	$\hat{C}_1 + \hat{D}_2 = 180° - 94°$ $\therefore \hat{D}_2 = 43°$
(b)(2)	$\hat{O}_2 = 360° - 94°$ ($\angle$s around a point) $\hat{O}_2 = 266°$ $\hat{B}_1 + \hat{B}_2 = \frac{266°}{2}$ ($\angle$ at centre) $\hat{B}_1 + \hat{B}_2 = 133°$	$\hat{O}_2 = 266°$ $\hat{B}_1 + \hat{B}_2 = \frac{266°}{2}$ ($\angle$ at centre) $\hat{B}_1 + \hat{B}_2 = 133°$
(b)(3)	$2x + 133° = 180°$ (int $\angle$s of Δ....) $x = 23\frac{1}{2}°$	$2x + 133° = 180°$ (int $\angle$s of Δ....) $x = 23\frac{1}{2}°$

QUESTION 6

(a)	$\hat{C}_1 = 90^\circ$ ($\angle$ in semi-circle)	$\hat{C}_1 = 90^\circ$ ($\angle$ in semi-circle)
(b)	$\hat{D} = 180^\circ - 38^\circ$ (opp. $\angle$s of cyclic quad) $\hat{D} = 142^\circ$	$\hat{D} = 142^\circ$ (opp. $\angle$s of cyclic quad)
(c)	$\hat{E}_1 = 180^\circ - (38^\circ + 90^\circ)$ (int. $\angle s$ of Δ) $\hat{E}_1 = 52^\circ$ $\hat{B} = 180^\circ - 52^\circ$ (opp. $\angle$s of cyclic quad) $\hat{B} = 128^\circ$	$\hat{E}_1 = 52^\circ$ $\hat{B} = 128^\circ$ (opp. $\angle$s of cyclic quad)
(d)	AF = FC (line from centre perp. to chord) $\therefore AF = 4$ $BC = 5$ (given) $\therefore BF = 3$ units (Pythagoras)	$AF = 4$ (line from centre perp. to chord) $\therefore BF = 3$ units

QUESTION 7

(a)	RTP: Area $\Delta PQR = \frac{1}{2}pq\sin\hat{R}$ Determine: *y*-coordinate of Q $\sin\hat{R} = \frac{y}{r}$ $y = r\sin\hat{R}$ $y = p\sin\hat{R}$ Area $\Delta PQR = \frac{1}{2}\text{base} \times \text{height}$ $= \frac{1}{2}q(p\sin\hat{R})$ $= \frac{1}{2}pq\sin\hat{R}$	sketch $\sin\hat{R} = \frac{y}{r}$ $y = p\sin\hat{R}$ Sub values in: $\Delta PQR = \frac{1}{2}\text{base} \times \text{height}$
(b)	Area $\Delta DBC = \frac{1}{2} \times 8 \times 8 \times \sin 60^\circ$ (equilateral Δ) $= 16\sqrt{3}$ Area of Prism $= 3 \times (15 \times 8) + 2 \times (16\sqrt{3})$ $= 415,4$ units	$\sin 60^\circ$ $3 \times (15 \times 8)$ $16\sqrt{3}$ $= 415,4$

SECTION B

QUESTION 8

(a)	$1-2\sin^2 x=-\frac{1}{7}$ for $[x\in -180°;90°]$ $\cos 2x=-\frac{1}{7}$ Ref. angle: $98,2°$ $2x=\pm 98,2°+k360°$ $(k\in Z)$ $x=\pm 49,1°+k180°$ $(k\in Z)$ $x\in\{-49,1°;49,1°;-130,9°\}$ **Alternate:** $1-2\sin^2 x=-\frac{1}{7}$ for $[x\in -180°;90°]$ $\sin x=\pm\sqrt{\frac{4}{7}}$ **, hence** $x=\pm 49,1°+k180°$ $(k\in Z)$ or $x=\pm 49,1°+k360°$ $(k\in Z)$ $x\in\{-49,1°;49,1°;-130,9°\}$	$\cos 2x=-\frac{1}{7}$ Ref. angle: $98,2°$ $2x=\pm 98,2°+k360°$ $(k\in Z)$ $x=\pm 49,1°+k180°$ $(k\in Z)$ $x\in\{-49,1°;49,1°;-130,9°\}$ $\sin x=\pm\sqrt{\frac{4}{7}}$ Ref. angle: $x=\pm 49,1°+k180°$ $(k\in Z)$ $x=\pm 49,1°+k360°$ $(k\in Z)$ $x\in\{-49,1°;49,1°;-130,9°\}$
(b)	$=(-\cos\theta)(-\sin^3\theta)-(-\tan\theta)(\cos\theta)(\cos^3\theta)$ $=\cos\theta.\sin^3\theta+\left(\frac{\sin\theta}{\cos\theta}\right)(\cos\theta)(\cos^3\theta)$ $=\cos\theta.\sin^3\theta+\sin\theta.\cos^3\theta$ $=\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)$ $=\sin\theta\cos\theta(1)$ $=\sin\theta\cos\theta$	$-\sin^3\theta$ $-\tan\theta$ $\cos^3\theta$ $\left(\frac{\sin\theta}{\cos\theta}\right)$ $=\sin\theta\cos\theta$

QUESTION 9

(a)	$(x+3)^2+(y-4)^2=25$ $C(-3;4) \quad r=5$	$(x+3)^2+(y-4)^2=25$ Completing the square $C(-3;4)$ $r=5$
(b)	For points A and B: sub. $x=2y-21$ into eq of circle $(2y-21)^2+y^2+6(2y-21)-8y=0$ $4y^2-84y+441+y^2+12y-126-8y=0$ $5y^2-80y+315=0$ $y^2-16y+63=0$ $(y-7)(y-9)=0$ $y=7$ or $y=9$ $\therefore B(x;7)$ $\therefore B(-7;7)$ $A(x;9)$ sub into equations $\therefore A(-3;9)$	$(2y-21)^2+y^2+$ $6(2y-21)-8y=0$ $y^2-16y+63=0$ $y=7$ or $y=9$ $B(-7;7)$ $A(-3;9)$

(c)(1)	For D: let $y=0$ $x^2+6x=0$ $x(x+6)=0$ $x=0$ or $x=-6$ $\therefore D(-6;0)$ and $A(-3;9)$ from (b) Midpt AD $\left(\frac{-3-6}{2};\frac{9+0}{2}\right)$ Midpt AD $\left(-\frac{9}{2};\frac{9}{2}\right)$	$D(-6;0)$ Midpt AD $\left(-\frac{9}{2};\frac{9}{2}\right)$
(c)(2)	Test for collinearity: If CB passes through the midpoint, then $m_{CB}=m_{CP}$ Using: $B(-7;7)$ and Centre$(-3;4)$ $m_{CB}=\frac{4-7}{-3+7}$ $m_{CB}=-\frac{3}{4}$ $m_{CP}=\frac{\frac{9}{2}-4}{-\frac{9}{2}+3}$ $m_{CP}=-\frac{1}{3}$ therefore, not collinear since: $m_{CB}\neq m_{CP}$ **Alternate:** Determine the equation of the straight-line BC: Using: $B(-7;7)$ and Centre$(-3;4)$ $m_{CB}=\frac{4-7}{-3+7}$ $m_{CB}=-\frac{3}{4}$ $y=-\frac{3}{4}x+c$ sub. pt $B(-7;7)$ or $C(-3;4)$ $c=\frac{7}{4}$ $y=-\frac{3}{4}x+\frac{7}{4}$ Sub. Midpt AD $\left(-\frac{9}{2};\frac{9}{2}\right)$ to test if AD lies on CD LHS = $\frac{9}{2}$ and RHS = $\frac{41}{8}$ LHS $\neq$ RHS therefore CB does not pass through the midpoint of line AD.	$m_{CB}=-\frac{3}{4}$ $m_{CP}=\frac{\frac{9}{2}-4}{-\frac{9}{2}+3}$ $m_{CB}\neq m_{CP}$ Conclusion Straight line equation to test $m_{CB}=-\frac{3}{4}$ $c=\frac{7}{4}$ LHS = $\frac{9}{2}$ Conclusion

(d)	Circle (1): $(x+3)^2+(y-4)^2=25$ Circle (2): $(x-3)^2+(y+4)^2=25$ Centre (1): $(-3;4)$ Centre (2): $(3;-4)$ Distance between centres: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=10$ units Sum of radii: 5 + 5 = 10 units The student is correct that the circles touch at a point since the distance between the centres is equal to the sum of the radii.	Centre (2): $(3;-4)$ Distance between centres $=10$ units Sum of radii: 5 + 5 = 10 units circles intersect/touch

QUESTION 10

In ΔDCF: DC = DF = 1,2 m Using cosine rule: $CF^2 = (1,2)^2 + (1,2)^2 - 2(1,2)(1,2)\cos 42^\circ$ $CF^2 = 0,7397$ m $CF = 0,86$ m In ΔADF: $(AF)^2 = (2,2)^2 + (1,2)^2$ (pythag) $AF = 2,506$ AF=AC In ΔACF: $\cos F\hat{A}C = \frac{(2,506)^2 + (2,506)^2 - (0,86)^2}{2(2,506)(2,506)}$ $\cos F\hat{A}C = 0,9411...$ $F\hat{A}C \approx 19,8$	DC = DF = 1,2 m $CF^2 = (1,2)^2 + (1,2)^2 -$ $2(1,2)(1,2)\cos 42^\circ$ $CF = 0,86$ $(AF)^2 = (2,2)^2 + (1,2)^2$ (pythag) $AF = 2,506$ $\cos F\hat{A}C =$ $\frac{(2,506)^2 + (2,506)^2 - (0,86)^2}{2(2,506)(2,506)}$ $F\hat{A}C \approx 19,8$
Alternate: In ΔDCF: DC = DF = 1,2 m $\therefore D\hat{C}F = \frac{180^\circ - 42^\circ}{2}$ ($\angle s$ opp = sides) $\therefore D\hat{C}F = 69^\circ$ $\frac{CF}{\sin 42^\circ} = \frac{1,2}{\sin 69^\circ}$ $CF = 0,86$ m In ΔADF: $(AF)^2 = (2,2)^2 + (1,2)^2$ (pythag) $AF = 2,506$ AF=AC In ΔACF: $\cos F\hat{A}C = \frac{(2,506)^2 + (2,506)^2 - (0,86)^2}{2(2,506)(2,506)}$ $\cos F\hat{A}C = 0,9411...$ $F\hat{A}C \approx 19,8$	DC = DF = 1,2 m $\therefore D\hat{C}F = \frac{180^\circ - 42^\circ}{2}$ ($\angle s$ opp = sides) $\therefore D\hat{C}F = 69^\circ$ $\frac{CF}{\sin 42^\circ} = \frac{1,2}{\sin 69^\circ}$ $CF = 0,86$ $(AF)^2 = (2,2)^2 + (1,2)^2$ (pythag) $AF = 2,506$ $\cos F\hat{A}C =$ $\frac{(2,506)^2 + (2,506)^2 - (0,86)^2}{2(2,506)(2,506)}$ $F\hat{A}C \approx 19,8$

QUESTION 11

(a)	$\dfrac{1+\sin 2x+\sin^2 x-\cos^2 x}{1+2\sin x.\cos x+\cos 2x}$ $=\dfrac{1+(2\sin x\cos x)+\sin^2 x-\cos^2 x}{1+2\sin x\cos x+(\cos^2 x-\sin^2 x)}$ $=\dfrac{\sin^2 x+\cos^2 x+2\sin x\cos x+\sin^2 x-\cos^2 x}{\cos^2 x+\sin^2 x+2\sin x\cos x+\cos^2 x-\sin^2 x}$ $=\dfrac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$ $=\dfrac{2\sin x(\sin x+\cos x)}{2\cos x(\cos x+\sin x)}$ $=\tan x$ $=\text{RHS}$	$(2\sin x\cos x)$ $(\cos^2 x-\sin^2 x)$ numerator: $\sin^2 x+\cos^2 x$ simplify $=\dfrac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$ factorise $=\dfrac{2\sin x(\sin x+\cos x)}{2\cos x(\cos x+\sin x)}$
(b)	Not valid for: $2\cos x(\cos x+\sin x)=0$ $2\cos x=0$ or $\cos x+\sin x=0$ For: $\cos x=0$ and tan x undefined: $\therefore x=\pm 90°+k360°$ $(k\in Z)$ Alternate: $x=90°+k180°$ $(k\in Z)$ or $\sin x=-\cos x$ $\dfrac{\sin x}{\cos x}=-1$ $\tan x=-1$ $x=-45°+k180°$ $(k\in Z)$ For tan: $x=90°+k180°$ $(k\in Z)$	$1+2\sin x\cos x+\cos 2x=0$ $\cos x=0$ and tan x is undefined $\cos x+\sin x=0$ $k\in Z$ General solutions

QUESTION 12

(a)	Prove: $\frac{CF}{FG}=\frac{GE}{EA}$ $\frac{CF}{FG}=\frac{CD}{DA}$ (line $\parallel$ one side of Δ) ;DF//AG $\frac{CD}{DA}=\frac{GE}{EA}$ (line $\parallel$ one side of Δ) ;$ED \parallel GC$ $\therefore \frac{CF}{FG}=\frac{GE}{EA}$	$\frac{CF}{FG}=\frac{CD}{DA}$ (line $\parallel$ one side of Δ) reason $\frac{CD}{DA}=\frac{GE}{EA}$ (line $\parallel$ one side of Δ) $\therefore \frac{CF}{FG}=\frac{GE}{EA}$
(b)	$\frac{CF}{FG}=\frac{2}{1}$ (given) $\therefore \frac{GE}{EA}=\frac{2}{1}$ but, $EA=\frac{1}{3}GA$ $\frac{EG}{\frac{1}{3}GA}=\frac{2}{1}$ $\therefore GE=\frac{2}{3}GA$ $\therefore GE:GA=2:3$	$\frac{GE}{EA}=\frac{2}{1}$ $\frac{EG}{\frac{1}{3}GA}=\frac{2}{1}$ $\therefore GE:GA=2:3$
(c)	$GE=\frac{2}{3}GA$ (proven) $GE=\frac{2}{3}\left(\frac{1}{2}AB\right)$ (G is the midpoint) $GE=\frac{1}{3}AB$ $\therefore DF=\frac{1}{3}AB$ $(DF=EG)$ $\therefore DF:AB=1:3$	$GE=\frac{2}{3}GA$ (proven) $GE=\frac{2}{3}\left(\frac{1}{2}AB\right)$ (G is the midpoint) $DF=EG$ $\therefore DF:AB=1:3$

QUESTION 13

(a)	Prove: ΔFEB is similar to ΔFGC In ΔFEB and ΔFGC $\hat{E}_2 = \hat{G}_1$ (given 90°) $\hat{B}_1 = \hat{C}_2$ (tan-chord th) $\therefore \Delta FEB \mathbin{///} \Delta FGC$ $(\angle;\angle;\angle)$	$\hat{E}_2 = \hat{G}_1$ (given) $\hat{B}_1 = \hat{C}_2$ (tan-chord th) $\therefore \Delta FEB \mathbin{///} \Delta FGC$ $(\angle;\angle;\angle)$
(b)	Prove: $FG^2 = FE \times FD$ In ΔFDC and ΔFGB $\hat{D}_2 = \hat{G}_2$ (given) $\hat{C}_1 = \hat{B}_2$ (tan-chord th) $\therefore \Delta FDC \mathbin{///} \Delta FGB$ $(\angle;\angle;\angle)$ $\frac{FD}{FG} = \frac{DC}{GB} = \frac{FC}{FB}$ (similar Δs) From (a): $\frac{FE}{FG} = \frac{EB}{GC} = \frac{FB}{FC}$ $\frac{FD}{FG} = \frac{FG}{FE}$ $\therefore FG^2 = FE \times FD$	$\hat{C}_1 = \hat{B}_2$ (tan-chord th) reason $\therefore \Delta FDC \mathbin{///} \Delta FGB$ $(\angle;\angle;\angle)$ $\frac{FD}{FG} = \frac{DC}{GB} = \frac{FC}{FB}$ (similar Δs) $\frac{FE}{FG} = \frac{EB}{GC} = \frac{FB}{FC}$ $\frac{FD}{FG} = \frac{FG}{FE}$ $\therefore FG^2 = FE \times FD$

QUESTION 14

(a)	Given: $2\cos 2\alpha + \sin 2\alpha = R\cos(2\alpha - \beta)$ RHS=$R(\sin 2\alpha \sin\beta + \cos 2\alpha \cos\beta)$ $= R\sin 2\alpha \sin\beta + R\cos 2\alpha \cos\beta$ $\therefore 2\cos 2\alpha + \sin 2\alpha = R\cos 2\alpha \cos\beta + R\sin 2\alpha \sin\beta$ $2\cos 2\alpha = R\cos 2\alpha \cos\beta$ $\therefore R\cos\beta = 2$ and, $\sin 2\alpha = R\sin 2\alpha \sin\beta$ $\therefore R\sin\beta = 1$ Square and Add: $R^2\cos^2\beta = 4$ and $R^2\sin^2\beta = 1$ $R^2(\cos^2\beta + \sin^2\beta) = 5$ $R^2 = 5$ $R = \sqrt{5}$ since $R > 0$ Solve for either: $R\sin\beta = 1$ and $R\cos\beta = 2$ $\sin\beta = \frac{1}{\sqrt{5}}$ $\therefore \beta = 26,6^\circ$ **Alternate:** Given: $2\cos 2\alpha + \sin 2\alpha = R\cos(2\alpha - \beta)$ RHS=$R(\sin 2\alpha \sin\beta + \cos 2\alpha \cos\beta)$ $= R\sin 2\alpha \sin\beta + R\cos 2\alpha \cos\beta$ $\therefore 2\cos 2\alpha + \sin 2\alpha = R\cos 2\alpha \cos\beta + R\sin 2\alpha \sin\beta$ $2\cos 2\alpha = R\cos 2\alpha \cos\beta$ $\therefore R\cos\beta = 2$ and, $\sin 2\alpha = R\sin 2\alpha \sin\beta$ $\therefore R\sin\beta = 1$ $\therefore \tan\beta = \frac{1}{2}$ $\beta = 26,6^\circ$ $\therefore R = \frac{2}{\cos 26,6^\circ} = 2,237 \approx 2,2$	$R(\sin 2\alpha \sin\beta + \cos 2\alpha \cos\beta)$ $R\cos\beta = 2$ $R\sin\beta = 1$ $R^2(\cos^2\beta + \sin^2\beta) = 5$ $R = \sqrt{5}$ $\sin\beta = \frac{1}{\sqrt{5}}$ $\beta = 26,6^\circ$ $R(\sin 2\alpha \sin\beta + \cos 2\alpha \cos\beta)$ $R\cos\beta = 2$ $R\sin\beta = 1$ $\therefore \tan\beta = \frac{1}{2}$ $\beta = 26,6^\circ$ $\therefore R = \frac{2}{\cos 26,6^\circ} = 2,237 \approx 2,2$

	Alternate: Given: $2\cos 2\alpha + \sin 2\alpha = \text{R}\cos(2\alpha - \beta)$ $\frac{2}{R}\cos 2\alpha + \frac{1}{R}\sin 2\alpha = \cos(2\alpha - \beta)$ $x^2 + y^2 = r^2$ $\therefore 2^2 + 1^2 = R^2$ $R = \sqrt{5}$ $2\cos 2\alpha + \sin 2\alpha = R\cos 2\alpha \cos\beta + R\sin 2\alpha \sin\beta$ $\sin 2\alpha = R\sin 2\alpha \sin\beta$ $1 = \sqrt{5}\sin\beta$ $\sin\beta = \frac{1}{\sqrt{5}} \quad \therefore \beta = 26{,}6°$	$\frac{2}{R}\cos 2\alpha + \frac{1}{R}\sin 2\alpha = \sin(2\alpha + \beta)$ $x^2 + y^2 = r^2$ $\therefore 2^2 + 1^2 = R^2$ $R^2 = 5$ $R = \sqrt{5}$ $\sin\beta = \frac{1}{\sqrt{5}}$ $\beta = 26{,}6°$
(b)	$4\cos^2\alpha + \sin 2\alpha$ $= 2(\cos 2\alpha + 1) + \sin 2\alpha$ $= 2\cos 2\alpha + \sin 2\alpha + 2$ Hence maximum is $\sqrt{5} + 2$	$= 2\cos 2\alpha + \sin 2\alpha + 2$ maximum is $\sqrt{5} + 2$

Total: 150 marks