NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2015

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) $M(6;\ 3)$ (2)

(b) $C\hat{M}A = 90^o$ $(Opp\ angles\ of\ cyclic\ quad)$

$m_{AB} = -\frac{6}{12}$

$\therefore m_{MC} = 2$

$y = 2x + c$

$sub\ in\ M(6;\ 3)$

$3 = 2(6) + c$

$c = -9$

$y = 2x - 9$ (5)

(c) (1) Area of Δ MCB = $\frac{1}{2}(b)(h)$

$2x - 9 = 0$

$x = 4\frac{1}{2}$

OB – OC = BC = 7,5

$h = 3$

$\therefore$ Area of ΔMCB= $\frac{1}{2}(7,5)(3)$ (4)

(2) $\therefore$ area of AMCO = 36 – 11,25

$= 24,75\ units^2$ (2)

[13]

QUESTION 2

(a) $\frac{1}{2}x + 4 = x$

$x + 8 = 2x$

$x = 8$

$\therefore y = x = 8$

$N(8;\ 8)$ (3)

(b) B(16; 16) (1)

(c) $7y = 10x$

$\therefore 7(16) = 10x$

$\therefore x_D = 11.2$

$D\ (11{,}2;16)$

$\therefore DB = 16 - 11{,}2$ (3)

$= 4{,}8\ units$

[7]

QUESTION 3

(a) (1) $= \dfrac{\sin\theta \sin\theta - 1}{\cos\theta}$

$= \dfrac{\sin^2\theta - 1}{\cos\theta}$

$= \dfrac{-\cos^2\theta}{\cos\theta}$

$= -\cos\theta$ (4)

(2) $-\cos\theta > 0 \ or \ \cos\theta < 0$

$\therefore \theta t \left(90°;\ 270°\right)$ (2)

(b) (1) $\tan\theta \sin\theta + \cos\theta$

$= \dfrac{\sin\theta}{\cos\theta}.\sin\theta + \cos\theta$

$= \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$ (3)

(2) $\dfrac{1}{\cos\theta} = \dfrac{3}{\sin\theta}$

$\sin\theta = 3\cos\theta$

$\tan\theta = 3$

$\text{Reference angle} = 71.57^o$

$\theta = 71.57° + k180° \qquad k \in Z$ (5)

[14]

QUESTION 4

$\hat{B} + \hat{D}_2 = 180°$	Opp angles of a cyclic quadrilateral
$\hat{D}_1 + \hat{D}_2 + \hat{D}_3 = 180°$	Angles on straight line
$\hat{D}_1 = \hat{C}_2$	tan chord theorem

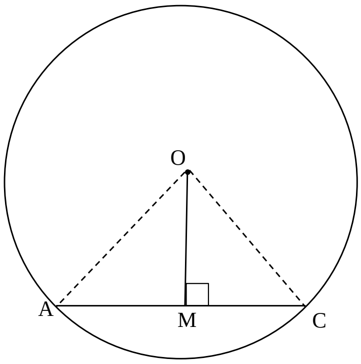
hence

$\hat{C}_2 + \hat{D}_3 = \hat{B}$ (3)

[3]

QUESTION 5

(a)



Construction: Join AO and OC
RTP: AM = MC
Proof: In Δ's OAM, OCM
OM is a common side
OA = OC (Radii)
$\hat{M}_1 = \hat{M}_2 = 90^o$ $(given)$
Therefore ΔAOM ≡ ΔCOM (R, H, S)
Hence AM = MC (6)

(b) (1) $\hat{B} = 90^o$ $(Angle\ in\ semi\ circle)$
$BE^2 = 20^2 - 12^2$ $(Pythag)$
$BE = 16\ units$ (2)

(2) $\frac{BC}{CE} = \frac{GO}{OE} = \frac{4}{10}\ or\ \frac{2}{5}$; line parallel to one side of Δ/mean proportion theorem (2)

(3) $BD = 8$ units $(Line\ \ from\ centre \perp chord)$
$\frac{BC}{16} = \frac{4}{14}$
$BC = 4{,}57$ units $OR\ \frac{32}{7}$ $= 4\frac{4}{7}$ units
$\therefore CD = 8 - 4{,}57$
$CD = 3{,}43$ units $OR\ \frac{24}{7}$ units (4)

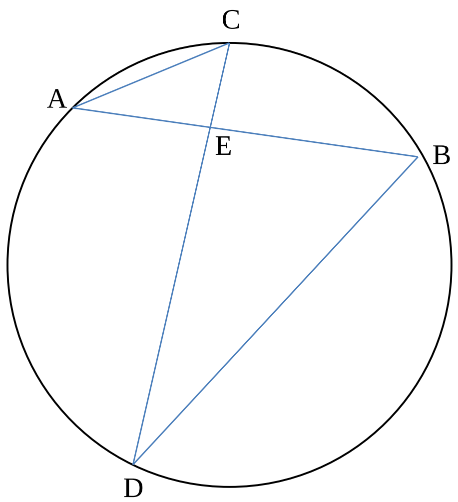
OR

Make BE = 1k; BC = $\frac{2}{7}k$ and $DE = \frac{1}{2}k$ $(line\ \ from\ centre \perp chord)$
therefore $CD = \left(1 - \frac{2}{7} - \frac{1}{2}\right) \times 16$
$CD = \frac{24}{7}$

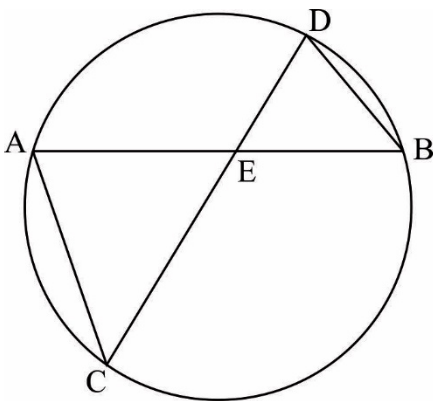
[14]

QUESTION 6

(a)



OR



(1)

(b) $\hat{A} = \hat{D}$ $(Angles\ in\ same\ seg)$

$\hat{C} = \hat{B}$ $(Angles\ in\ same\ seg)$

$\Delta AEC\ ///\ \Delta DEB$ (AAA) or they could give the third angle (3)

(c) $\frac{AE}{EC} = \frac{DE}{EB}$ $(\Delta AEC\ ///\ \Delta DEB)$

$\therefore \text{AE} \times \text{EB} = \text{DE} \times \text{EC}$ (2)

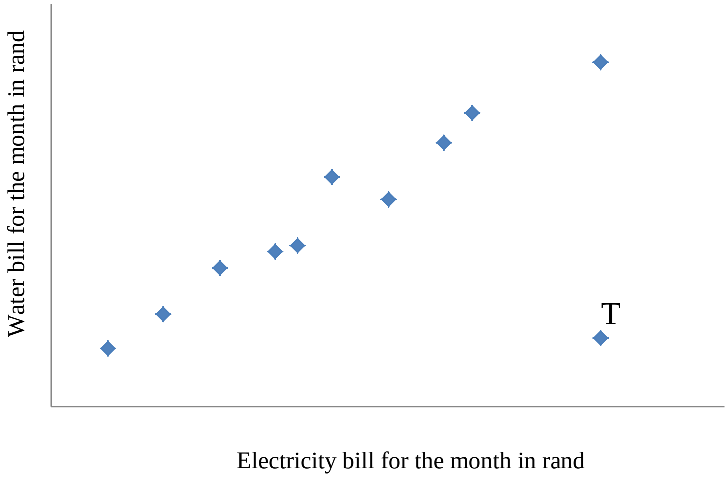
[6]

QUESTION 7

(a) (1) $y = 4x - 2$ (1 for line equation; 1 for the gradient and 1 for the y intercept) (3)

(2) r = 1 perfect correlation (2)

(b) (1)



(1)

(2) Strong/positive relationship or (as the water bill increases so does the elec bill) (1)

(3) It would increase. (1)

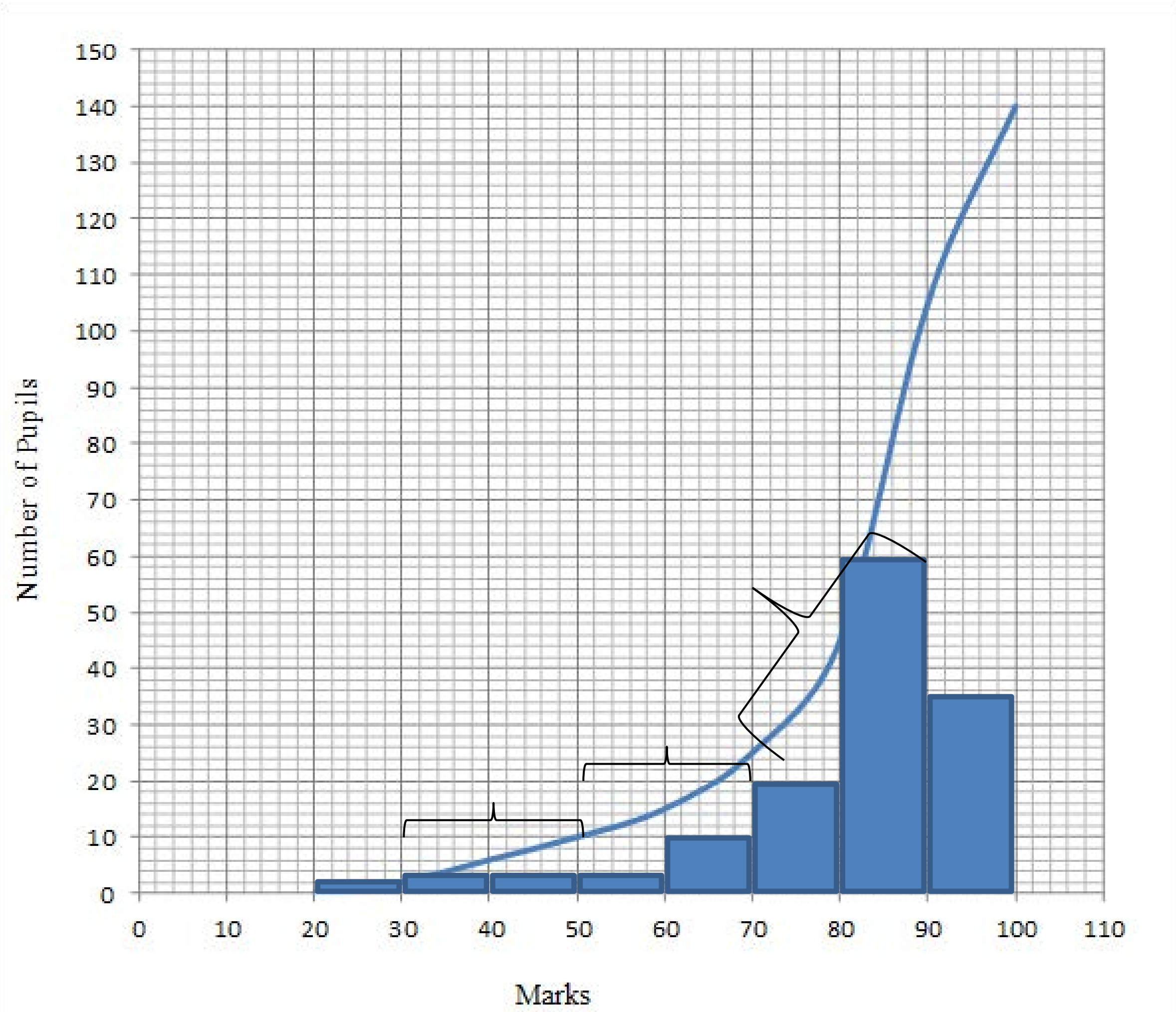
(4) B would increase. (1)

(5) No, as you would be extrapolating.
OR: the equation is valid for values of x between 100 and 1 000
OR: Yes, but the result will be innacurate (1)

[10]

QUESTION 8

(a)



For recognising it is a histogram e.g. no spaces (5)

(b) The mark distribution is skewed to the left or negatively skewed (1)

(c) TRUE; Data skewed to left (2)

Or they calculate the quartile values and make decision: 75; 85 and 90

[8]

SECTION B

QUESTION 9

(a) $\hat{B}+\hat{C}=\hat{D}+\hat{F}$ (1)

(b) $\hat{B}_1=\hat{D}_2$ $(Alternate\ angles\ AB//DC)$

$\hat{C}=\hat{A}$ $(Angle\ subtended\ from\ equal\ chord\ of\ equal\ circle)$

$\therefore \hat{B}_2=\hat{D}_1$ $(Angles\ in\ a\ triangle)$

But these are alternate angles

$BC//AD\ and\ ABCD$ is a parallelogram (6)

OR:

$\hat{B}_1=\hat{D}_2$ (alternate angles AB//DC) ✓

$\hat{C}=\hat{A}$ (angle subtended from equal chord of equal circle)

BD is a common side

$\therefore \Delta ABD \equiv \Delta CBD$ (A.S.A) ✓

AB = CD

$\therefore$ *ABCD* is a parallelogram (one pair of opposite sides equal and parallel)

OR

$\hat{B}_1+\hat{B}_2=180-\hat{C}$ $(co-\text{int}\ angles\ //\ lines)$

$\hat{D}_1+\hat{D}_2=180-\hat{A}$ $(co-\text{int}\ //\ lines)$

but

$\hat{A}=\hat{C}$ $(Angle\ subtended\ by\ same\ chord\ equal\ circles)$

$\therefore \hat{B}_1+\hat{B}_2=\hat{D}_1+\hat{D}_2$

$\therefore$ *parm* (*Opp angles are equal*)

[7]

QUESTION 10

(a) $\hat{C} = \hat{B} = 55°$ (tan chord theorem) OR (angles in same segment)

$\hat{D}_2 = 18°$ (tan chord theorem)

$\hat{D}_1 + \hat{D}_2 = 55°$ (alt angles CD // tangent)

$\therefore \hat{D}_1 = 37°$

$\hat{E}_2 = 180° - (55° + 37°)$ ($\angle$'s in a $\triangle$)

$= 88°$ (6)

(b) (1) $P\hat{D}O = 90°$ (line from centre $\perp$ tangent)

$P\hat{E}O = 90°$ (line from centre $\perp$ tangent)

$\hat{P} = 2\hat{A}$ ($\angle$ at centre = 2 × $\angle$ at circumference)

$\hat{O}_1 = 180° - 2\hat{A}$ (Opp $\angle$'s of quad) (8)

(2) $\hat{O}_2 = 180° + 2\hat{A}$ ($\angle$ around a point)

$\hat{K}_2 = 90° + \hat{A}$ ($\angle$ at centre = 2 × $\angle$ at circumference)

$\hat{K}_2 = \hat{C}_3 + \hat{E}_1$ (ext $\angle$ of $\triangle$ = sum of interior opposite angles)

$\therefore \hat{C}_3 + \hat{E}_1 = 90 + \hat{A}$

Alternate:

Construct line DE.

$\hat{C}_3 + \hat{E}_1 = K_2$ (ext $\angle$ of $\triangle$ = sum of interior opposite angles)

$\hat{K}_2 = 180° - (C\hat{D}E + K\hat{E}D)$ ($\angle$'s in $\triangle$)

but $\hat{E}_1 = C\hat{D}E$ (tan chord theorem)

$\therefore \hat{K}_2 = 180° - (\hat{E}_1 + K\hat{E}D)$

but $\hat{E}_1 = K\hat{E}D = \dfrac{180 - \hat{P}}{2}$ (PD = PE, tangents from common point)

$\therefore \hat{C}_3 + \hat{E}_1 = 180° - \left(90 - \dfrac{\hat{P}}{2}\right)$

$= 90° + \hat{A}$ (6)

[20]

QUESTION 11

(a) $2\sin 2x + 2 = 2\cos x + 2$ $\quad 2\sin 2x = 2\cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \; or \; \sin x = \frac{1}{2}$

$x = \pm 90° + k360° \;\; or \;\; x = 30° + k360° \;\; or \;\; 150° + k360°$

$\therefore A(150°; -\sqrt{3} + 2)$

Note:
Calculator Answers:
Equ 1 = Equ 2 plus answer 7/7
Answer Only 2/7
Equ 1 = Equ 2 x = 30 1/7

OR $2\sin 2x + 2 = 2\cos x + 2$ $\quad 2\sin 2x = 2\cos x$

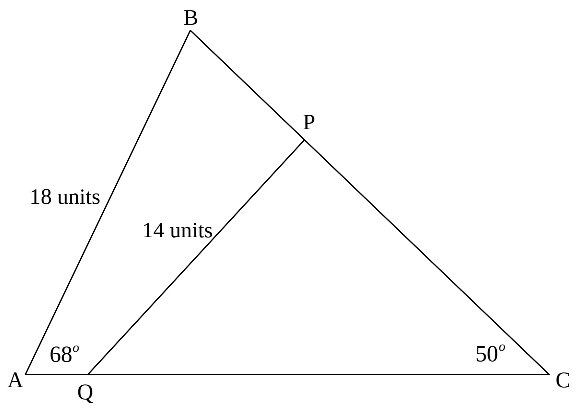
$\sin 2x = \cos x$

$\sin 2x = \sin(90° - x)$ $\quad \sin 2x = \sin(90° + x)$ $\quad 2x = 90 - x$

$x = 30° + k.120°$ $\quad x = 90° + k.360°$

$\therefore A(150°; -\sqrt{3} + 2)$ (7)

(b)



$\frac{BC}{\sin 68°} = \frac{18}{\sin 50°}$

$BC = \frac{18\sin 68°}{\sin 50°}$

$PC = BC \times \frac{3}{5}$

$\frac{\sin P\hat{Q}C}{PC} = \frac{\sin 50}{14}$

$\sin P\hat{Q}C = \frac{18\sin 68°}{\sin 50°} \times \frac{3}{5} \times \sin 50° \times \frac{1}{14}$

$\sin P\hat{Q}C = \frac{54\sin 68°}{70}$

$P\hat{Q}C = 45,66°$ (6)

(c)

$$\frac{\cos A\cos 45° + \sin A\sin 45°}{\cos A\cos 45° - \sin A\sin 45°}$$

$$\frac{\frac{1}{\sqrt{2}}(\cos A + \sin A)}{\frac{1}{\sqrt{2}}(\cos A - \sin A)}$$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$\frac{\cos^2 A + 2\sin A\cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$$

$$\frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)}$$

$\therefore$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)}$$

OR

$$\frac{\cos A\cos 45° + \sin A\sin 45°}{\cos A\cos 45° - \sin A\sin 45°}$$

$$\frac{\frac{1}{\sqrt{2}}(\cos A + \sin A)}{\frac{1}{\sqrt{2}}(\cos A - \sin A)}$$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)} \times \frac{(\cos A + \sin A)}{(\cos A + \sin A)}$$

$$\frac{\cos^2 A + 2\sin A\cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$$

$$\frac{1 + \sin 2A}{\cos 2A}$$

(6)
[19]

QUESTION 12

(a) $x^2 + 10x + 25 + y^2 - 6y + 9 = 30 + 25 + 9$

$(x+5)^2 + (y-3)^2 = 64$

$\therefore$ *Radius is* 8 *units* (4)

(b) PQ = $\sqrt{\left(7-(-5)\right)^2 + \left(-2-3\right)^2}$

PQ = $\sqrt{169}$

PQ = 13 (3)

(c) P(7; –2)

Q(–5; 3)

$M_{PQ} = \dfrac{-2-3}{7-(-5)}$

$= \dfrac{-5}{12}$

$y = \dfrac{-5}{12}x + c$

sub in (7; –2)

$-2 = \dfrac{-5}{12}(7) + c$

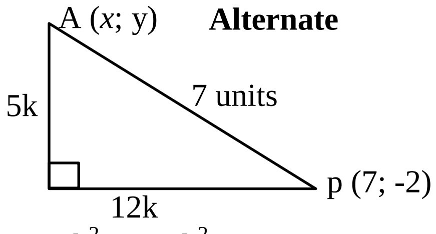
$c = \dfrac{11}{12}$

$\therefore\ y = \dfrac{-5x}{12} + \dfrac{11}{12}$

$12y = -5x + 11$

$\therefore\ 5x + 12y = 11$ (4)

(d) **Alternate**



$25k^2 + 144k^2 = 49$

$k^2 = \dfrac{49}{169}$

$k = \dfrac{7}{13}$

$A\left(7 - 12\left(\dfrac{7}{13}\right); -2 + 5\left(\dfrac{7}{13}\right)\right)$

$A\ \dfrac{-61}{13}$

Main

$(x-7)^2 + (y+2)^2 = 49$
$5x + 12y = 11$
Solve simultaneously

$169y^2 + 676y - 549 = 0$

$y = \dfrac{9}{13}$ OR $\dfrac{-61}{13}$

$5x + 12\left(\dfrac{9}{13}\right) = 11$

$x = \dfrac{7}{13}$

$A\left(\dfrac{7}{13}; \dfrac{9}{13}\right)$

(5)

OR

$(x-7)^2+(y+2)^2=49$
$5x+12y=11$ (Recognising line and circle)

Substitution str line int*o cirlce*

$169y^2 + 676y - 549 = 0$

Substituting the correct value into the equation to get other coordinate

$A\left(\frac{7}{13};\frac{9}{13}\right)$

(e) $\left(x-7\right)^2+\left(y+2\right)^2-49=x^2+y^2+10x-6y-30$

$x^2-14x+49+y^2+4y+4-49=x^2+y^2+10x-6y-30$

$10y-24x=-34$

OR $y=\frac{12x}{5}-\frac{17}{5}$ (3)

(f) $\frac{12}{5}\times\frac{-5}{12}=-1$

$\therefore$ perpendicular

OR

PCQD is a kite
$\therefore$ diagonals are perpendicular to each other. (2)

[21]

QUESTION 13

$BC^2 = 1^2 + 1,5^2 - 2(1)(1,5)\cos 30°$

$BC = 0,8074179764$

$Area\ of\ triangle = \frac{1}{2}(1)(1,5)\sin 30°$

$= 0,375\, m^2$

$\frac{1}{2} \times BC \times h = 0,375$

$h = 0,9288869234$

Therefore

Volume of the water tank is:

$\pi(3)^2 \times 0,9288869234$

$= \underline{\underline{26,26\ m^3}}$

Alternate method of finding height
$\frac{80,7}{\sin 30} = \frac{100}{\sin B\hat{C}A}$
$B\hat{C}A = 38.26$
Use trig ratio
$\sin 38,26 = \frac{h}{150}$
$h = 92,89\, cm$

[8]

Total: 150 marks