



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2014

MEMORANDUM

MARKS/PUNTE: 150

**This memorandum consists of 23 pages.
*Hierdie memorandum bestaan uit 23 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.
- Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.

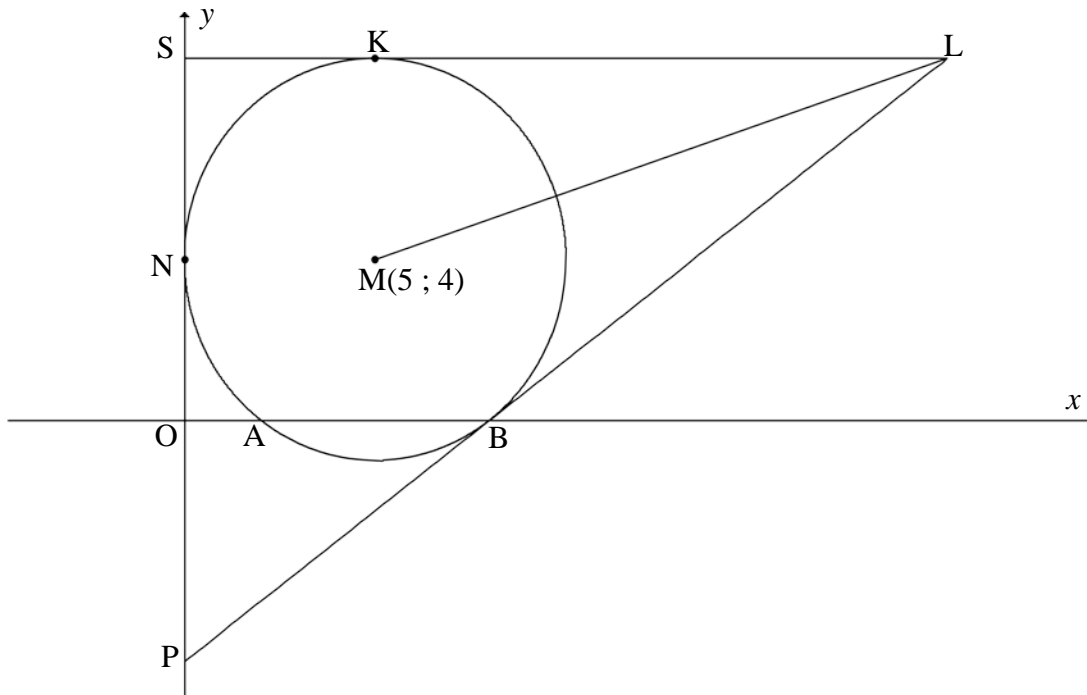
QUESTION/VRAAG 1

1.1	$\bar{x} = \frac{816}{12} = 68$	✓ $\frac{816}{12}$ ✓ 68 (2)
1.2	$\sigma = 18,42$	✓ answer/antw (1)
1.3	$(68 - 18,42 ; 68 + 18,42) = (49,58 ; 86,42)$ \therefore 6 candidates had a mark within one standard deviation of the mean/6 kandidate het 'n punt binne een standaardafwyking vanaf die gemiddelde.	✓✓ interval ✓ answer/antw (3)
1.4	$a = 22,828\dots = 22,83$ $b = 0,66429\dots = 0,66$ $\therefore \hat{y} = 0,66x + 22,83$ OR/OF $\hat{y} = 22,83 + 0,66x$	✓ value of a/ waarde van a ✓ value of b/ waarde van b ✓ equation/vgl (3)
1.5	$\hat{y} = 0,66x + 22,83$ $y = 0,66(60) + 22,83$ $62,43\dots\% \approx 62\%$ OR/OF $62,69\% \approx 63\%$	✓ subs of 60 into equation ✓ answer/antw (2) ✓✓ answer/antw (2)
1.6	(82 ; 62)	✓ answer/antw (1) [12]

QUESTION/VRAAG 2

2.1	$50 < x \leq 60$ OR/OF $50 \leq x < 60$ OR/OF between 50 and 60/tussen 50 en 60			✓ answer/antw (1)																											
2.2.1	<table border="1"> <thead> <tr> <th>Class <i>Klas</i></th> <th>Frequency <i>Frekwensie</i></th> <th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$20 < x \leq 30$</td> <td>1</td> <td>1</td> </tr> <tr> <td>$30 < x \leq 40$</td> <td>7</td> <td>8</td> </tr> <tr> <td>$40 < x \leq 50$</td> <td>13</td> <td>21</td> </tr> <tr> <td>$50 < x \leq 60$</td> <td>17</td> <td>38</td> </tr> <tr> <td>$60 < x \leq 70$</td> <td>9</td> <td>47</td> </tr> <tr> <td>$70 < x \leq 80$</td> <td>5</td> <td>52</td> </tr> <tr> <td>$80 < x \leq 90$</td> <td>2</td> <td>54</td> </tr> <tr> <td>$90 < x \leq 100$</td> <td>1</td> <td>55</td> </tr> </tbody> </table>			Class <i>Klas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$20 < x \leq 30$	1	1	$30 < x \leq 40$	7	8	$40 < x \leq 50$	13	21	$50 < x \leq 60$	17	38	$60 < x \leq 70$	9	47	$70 < x \leq 80$	5	52	$80 < x \leq 90$	2	54	$90 < x \leq 100$	1	55	✓ 8 ✓ 55 (2)
Class <i>Klas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																													
$20 < x \leq 30$	1	1																													
$30 < x \leq 40$	7	8																													
$40 < x \leq 50$	13	21																													
$50 < x \leq 60$	17	38																													
$60 < x \leq 70$	9	47																													
$70 < x \leq 80$	5	52																													
$80 < x \leq 90$	2	54																													
$90 < x \leq 100$	1	55																													
2.2.2				✓ grounding at (20 ; 0)/ anker by (20 ; 0) ✓ plotting at upper limits/ plot by boonste limiete ✓ smooth shape of curve/gladde kurwe (3)																											
2.3	55 – 44 (accept/aanvaar 43 – 45) ≈ 11 motorists/motoriste (accept/aanvaar 10 – 12 motorists/motoriste)			✓ 44 ✓ 11 (2) [8]																											

QUESTION/VRAAG 3

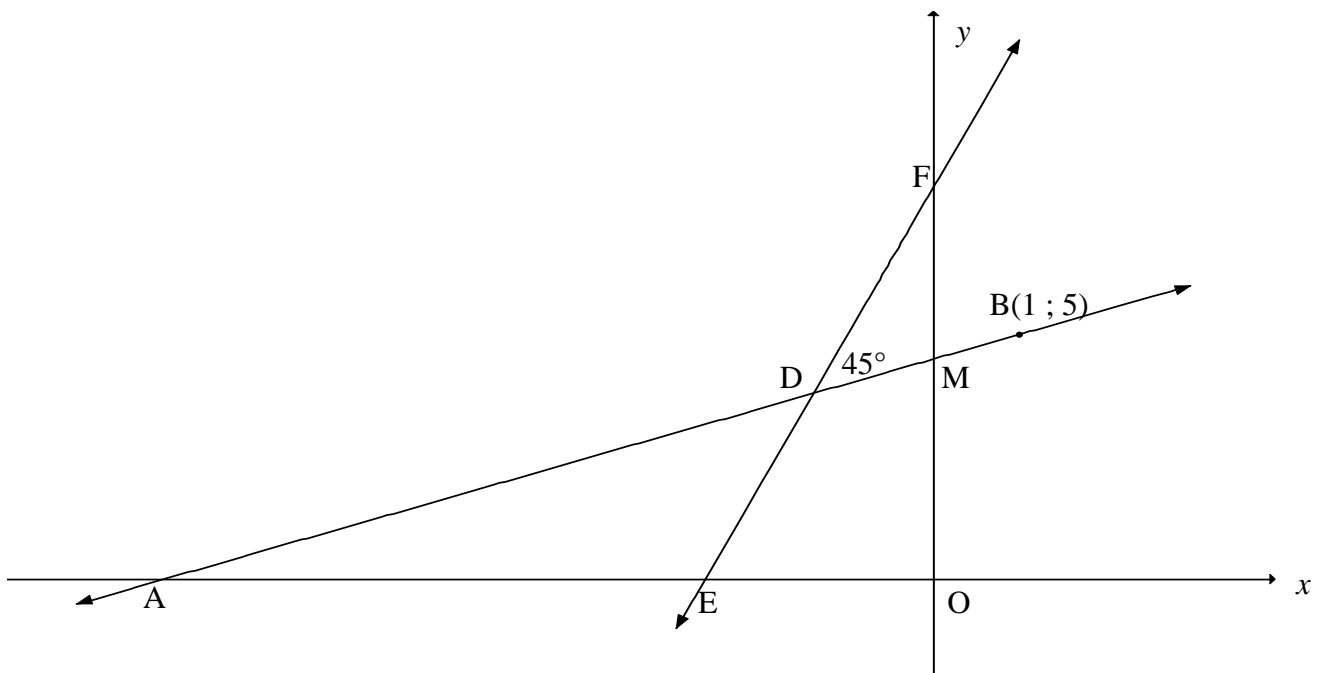


3.1	$r = MN = 5$	✓ answer/antw (1)	
3.2	$(x - 5)^2 + (y - 4)^2 = 25$	✓ equation/vgl (1)	
3.3	$A(x ; 0)$ $(x - 5)^2 + (0 - 4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x - 8)(x - 2) = 0$ $\therefore x = 8$ or/of $x = 2$ $\therefore A(2 ; 0)$	$(x - 5)^2 + (0 - 4)^2 = 25$ $(x - 5)^2 + 16 = 25$ $(x - 5)^2 = 9$ $(x - 5) = \pm 3$ $\therefore x = 8$ or/of $x = 2$ $\therefore A(2 ; 0)$	✓ substitute into eq/ vervang in vgl $y = 0$ ✓ standard form/ standaardvorm or perfect square form/kwadr vorm ✓ answer/antw (3)
3.4.1	$m_{MB} = \frac{4 - 0}{5 - 8}$ $= -\frac{4}{3}$	✓ subst M and B into form/vervang M and B in form ✓ $m_{MB} = -\frac{4}{3}$ (2)	

<p>3.4.2</p>	<p>$m_{MB} \times m_{PB} = -1$ (tangent \perp radius/ $rkl \perp$ radius) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ OR/OF $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$</p>	<p>✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/vgl (3)</p>
<p>3.5</p>	<p>$y_K = y_M + r = 4 + 5$ $y = 9$</p>	<p>✓ 9 ✓ equation/vgl (2)</p>
<p>3.6</p>	<p>At/By L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore L(20 ; 9)$</p>	<p>✓ equating simultaneously ✓ simplification (2)</p>
<p>3.7</p>	<p>$L(20 ; 9)$ $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ OR/OF $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20 - 5)^2 + (9 - 4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9 + 1)}$ $= \sqrt{250}$ or / of $5\sqrt{10}$ $= \sqrt{250}$ or / of $5\sqrt{10}$</p>	<p>✓ correct subst into distance formula/ korrekte subst in afstand- formule ✓ answer in surd form/antw in wortelvorm (2)</p>
<p>3.8</p>	<p>MK \perp KL OR/OF $\widehat{MKL} = 90^\circ$ (radius \perp tangent/radius \perp rkl) $\therefore ML$ is a diameter as it subtends a right angle/ML is middellyn $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/Midpt van sirkel = midpt v ML $x = \frac{5 + 20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4 + 9}{2} = \frac{13}{2} = 6,5$ Centre/midpt: (12,5 ; 6,5) Equation of the circle KLM /Vgl van sirkel KLM: $\therefore (x - 12,5)^2 + (y - 6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$ OR/OF</p>	<p>✓ S ✓ value of/waarde van r ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ answer in correct form/ antw in korrekte vorm (5)</p>

<p>MK ⊥ KL OR/OF $\widehat{MKL} = 90^\circ$ (radius ⊥ tangent/radius ⊥ rkl) \therefore ML is a diameter as it subtends a right angle/ML is middellyn Centre of circle = midpoint of ML/Midpt van sirkel = midpt v ML $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/midpt: (12,5 ; 6,5) Equation of the circle KLM /Vgl van sirkel KLM: $(x-12,5)^2 + (y-6,5)^2 = r^2$ subst (5 ; 4): $(5-12,5)^2 + (4-6,5)^2 = r^2$ $62,5 = r^2$ $\therefore (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$</p> <p>OR/OF</p> <p>By symmetry about LM/deur simmetrie om LM: MK ⊥ KL OR/OF $\widehat{MKL} = 90^\circ$ (radius ⊥ tangent/radius ⊥ rkl) \therefore ML is a diameter as it subtends a right angle/ML is middellyn ML is a diameter /ML is 'n middellyn $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/Midpt van sirkel = midpt v ML $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/midpt: (12,5 ; 6,5) Equation of the circle KLM /Vgl van sirkel KLM: $\therefore (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$</p>	<p>✓ S</p> <p>✓ $x = 12,5$ ✓ $y = 6,5$</p> <p>✓ value of/waarde van r^2</p> <p>✓ answer in correct form/antw in korrekte vorm (5)</p> <p>✓ S</p> <p>✓ value of/waarde van r</p> <p>✓ $x = 12,5$ ✓ $y = 6,5$</p> <p>✓ answer in correct form/antw in korrekte vorm (5)</p> <p>[21]</p>
--	--

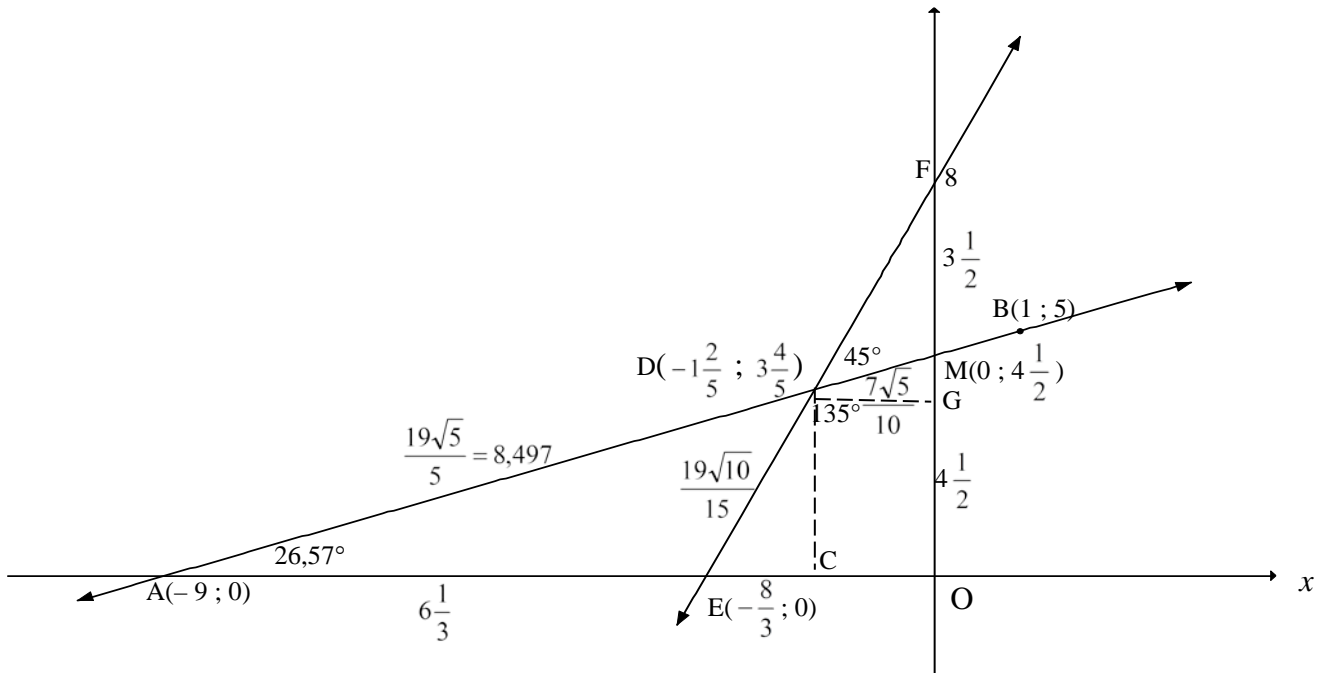
QUESTION/VRAAG 4



4.1	$y = 0: 3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore E\left(-2\frac{2}{3}; 0\right) \text{ OR/OF } E\left(-\frac{8}{3}; 0\right)$	✓ <i>y-value/waarde</i> ✓ <i>x-value/waarde</i> (2)
4.2	$\tan \hat{D}EO = m_{DE} = 3$ $\therefore \hat{D}EO = 71,565\dots = 71,57^\circ$ $\hat{D}AE = 71,565\dots^\circ - 45^\circ$ $= 26,57^\circ$	✓ $\tan \hat{D}EO = 3$ ✓ $71,565\dots^\circ$ ✓ $26,57^\circ$ (3)
4.3	$m_{AB} = \tan 26,57^\circ$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c \quad \text{OR/OF} \quad y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c \quad y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2} \quad y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{AB} = \tan 26,57^\circ$ ✓ $m_{AB} = \frac{1}{2}$ ✓ subst of m and $(1; 5)$ into formula/ <i>subst m en $(1; 5)$ in formule</i> ✓ equation/vgl (4)

<p>4.4</p>	<p>Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously:</p> $x - 2(3x+8) + 9 = 0$ $x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$ <p>$\therefore y = 3(-1\frac{2}{5}) + 8$ OR/OF $-1\frac{2}{5} - 2y + 9 = 0$</p> $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ <p>$\therefore D(-1\frac{2}{5} ; 3\frac{4}{5})$</p> <p>OR/OF</p> $x = 2y - 9$ $y = 3(2y - 9) + 8$ $y = 6y - 27 + 8$ <p>$\therefore y = 3\frac{4}{5}$</p> $x = 2(3\frac{4}{5}) - 9$ <p>OR/OF $3\frac{4}{5} = 3x + 8$</p> $x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$ <p>$\therefore D(-1\frac{2}{5} ; 3\frac{4}{5})$</p> <p>OR/OF</p> $3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$ $6x + 16 = x + 9$ $5x = -7$ <p>$\therefore x = -1\frac{2}{5}$</p> <p>$\therefore y = 3(-1\frac{2}{5}) + 8$ OR/OF $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$</p> $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ <p>$\therefore D(-1\frac{2}{5} ; 3\frac{4}{5})$</p> <p>OR/OF</p>	<p>✓ subst/vervang</p> <p>✓ x-value/waarde</p> <p>✓ subst/vervang</p> <p>✓ y-value/waarde (4)</p> <p>✓ subst/vervang</p> <p>✓ y value/waarde</p> <p>✓ subst/vervang</p> <p>✓ x-value/waarde</p> <p>(4)</p> <p>✓ equating/gelyk stel</p> <p>✓ x value/waarde</p> <p>✓ subst/vervang</p> <p>✓ y-value/waarde (4)</p>
------------	---	--

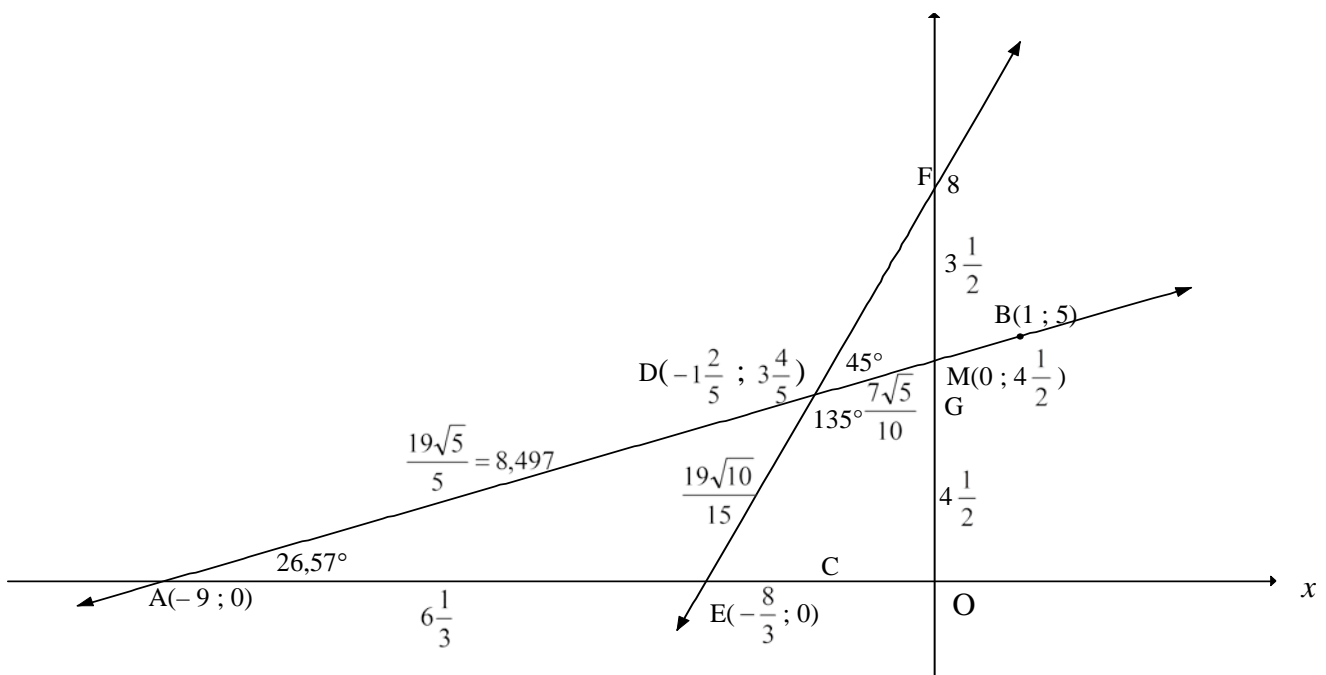
	$x - 2y = -9 \dots\dots(1)$ $-6x + 2y = 16 \dots\dots(2)$ <p>(1) + (2):</p> $-5x = 7$ $\therefore x = -1\frac{2}{5}$ $\therefore -1\frac{2}{5} - 2y = -9 \quad \text{OR/OF} \quad y = 3(-1\frac{2}{5}) + 8$ $y = 3\frac{4}{5} \qquad y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$ <p>OR/OF</p> $y = 3x + 8 \dots\dots\dots(1)$ $6y = 3x + 27 \dots\dots\dots(2)$ <p>(1) - (2):</p> $-5y = -19$ $\therefore y = 3\frac{4}{5}$ $3\frac{4}{5} = 3x + 8 \qquad \text{OR/OF} \qquad x = 2(3\frac{4}{5}) - 9$ $x = -1\frac{2}{5} \qquad x = -1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$	<p>✓ adding/optelling</p> <p>✓ x-value/waarde</p> <p>✓ subst/vervang</p> <p>✓ y-value/waarde</p> <p style="text-align: right;">(4)</p> <p>✓ subtracting/aftrekking</p> <p>✓ y-value/waarde</p> <p>✓ subst/vervang</p> <p>✓ x-value/waarde</p> <p style="text-align: right;">(4)</p>
--	--	---



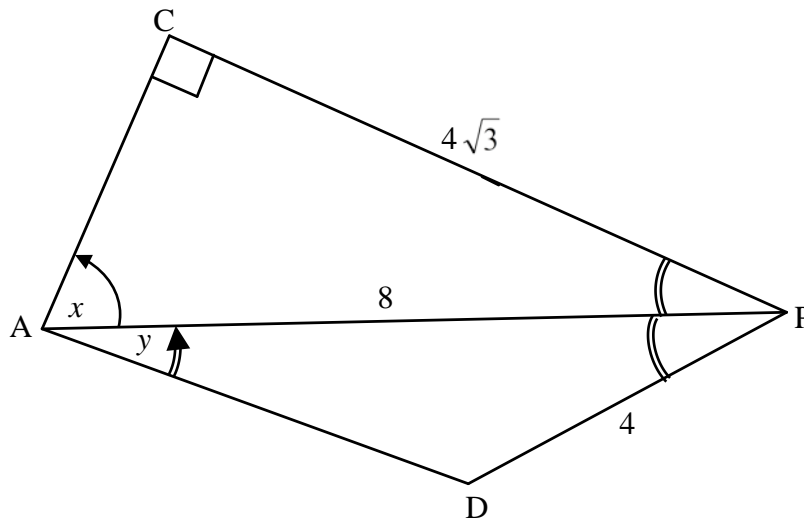
<p>4.5</p> <p>area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore A(-9; 0)$</p> <p>area ΔAMO $= \frac{1}{2} \cdot AO \cdot OM$ $= \frac{1}{2} (9)(4 \frac{1}{2})$ $= 20,25$</p> <p>area ΔADE $= \frac{1}{2} \cdot AE \cdot y_D$ $= \frac{1}{2} \cdot (AO - EO) \cdot y_D$ $= \frac{1}{2} \left(9 - 2 \frac{2}{3} \right) \left(3 \frac{4}{5} \right)$ $= 12,03$</p> <p style="text-align: center;">OR/OF</p> <p>area ΔADE $= \frac{1}{2} AD \cdot AE \cdot \sin \hat{D}AE$ $= \frac{1}{2} \left(\frac{19\sqrt{5}}{5} \right) \cdot 6 \frac{1}{3} \cdot \sin 26,57^\circ$ $= 12,03$</p> <p>\therefore area DMOE = 8,22 square units/vk eenh</p> <p style="text-align: center;">OR/OF</p>	<p>✓ correct method/ korrekte metode</p> <p>✓ $x_A = -9$</p> <p>✓ $\frac{1}{2} (9)(4 \frac{1}{2})$</p> <p>✓ $AE = 9 - 2 \frac{2}{3} = 6 \frac{1}{3}$</p> <p>✓ $y_D = 3 \frac{4}{5}$</p> <p style="text-align: center;">OR/OF</p> <p>✓ $AD = \frac{19\sqrt{5}}{5}$</p> <p>✓ $AE = 6 \frac{1}{3}$</p> <p>✓ answer/antw</p> <p style="text-align: right;">(6)</p>
--	---

	<p>area DMOE = area rectangle DCOG + area $\triangle DMG$ + area $\triangle DEC$</p> $= \left(1\frac{2}{5} \times 3\frac{4}{5}\right) + \frac{1}{2}\left(1\frac{2}{5}\right)\left(\frac{7}{10}\right) + \frac{1}{2}\left(3\frac{4}{5}\right)\left(\frac{19}{15}\right)$ $= 8,22 \text{ square units/vk eenh}$ <p style="text-align: center;">OR/OF</p> <p>area DMOE = area $\triangle EDO$ + area $\triangle ODM$</p> $= \frac{1}{2}(\text{EO} \times y_D) + \frac{1}{2}(\text{OM} \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304 + 189}{30}\right)$ $= \frac{493}{60} \text{ or/of } 8\frac{13}{60} \text{ or/of } 8,22 \text{ square units/vk eenh}$ <p style="text-align: center;">OR/OF</p> <p>area DMOE = area $\triangle EOF$ – area $\triangle DMF$</p> $= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640 - 147}{30}\right)$ $= \frac{493}{60} \text{ or } 8\frac{13}{60} \text{ or } 8,22 \text{ square units/vk eenh}$ <p style="text-align: center;">OR/OF</p>	<p>✓ correct method/ korrekte metode</p> <p>✓ $3\frac{4}{5}$</p> <p>✓ $1\frac{2}{5}$ ✓ 0,7</p> <p>✓ $\frac{19}{15}$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p> <p>✓ correct method/ korrekte metode</p> <p>✓ $y_D = \frac{19}{5}$ or $3\frac{4}{5}$</p> <p>✓ $\text{EO} = \frac{8}{3}$</p> <p>✓ $-x_D = \frac{7}{5}$</p> <p>✓ $\text{OM} = \frac{9}{2}$ or $4\frac{1}{2}$</p> <p>✓ answer/antw</p> <p style="text-align: right;">(6)</p> <p>✓ correct method/ korrekte metode</p> <p>✓ $y_F = 8$</p> <p>✓ $\text{EO} = \frac{8}{3}$</p> <p>✓ $-x_D = \frac{7}{5}$</p> <p>✓ $\text{FM} = 3\frac{1}{2}$</p> <p>✓ answer/antw</p> <p style="text-align: right;">(6)</p>
--	--	--

<p>area $\Delta EOM = \frac{1}{2}(EO \times OM)$</p> $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $= 6 \text{ sq units/vk eenh}$ <p>$ED = \sqrt{\left(-\frac{7}{5} + \frac{8}{3}\right)^2 + \left(\frac{19}{5}\right)^2}$ and $DM = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{9}{2} - \frac{19}{5}\right)^2}$</p> $= \frac{19\sqrt{10}}{15} \text{ or } 4,005\dots$ $= \frac{7\sqrt{5}}{10} \text{ or } 1,565\dots$ <p>area $\Delta EDM = \frac{1}{2}(ED \times DM \times \sin \widehat{EDM})$</p> $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135^\circ$ $= \frac{133}{60} \text{ or } 2,216\dots$ <p>\therefore area DMOE = area ΔEOM + area ΔEDM</p> $= 6 + 2,216\dots$ $= \frac{493}{60} \text{ or/of } 8\frac{13}{60} \text{ or/of } 8,22 \text{ square units/eenh}^2$	<p>✓ area ΔEOM</p> <p>✓ $ED = \frac{19\sqrt{10}}{15}$</p> <p>✓ $DM = \frac{7\sqrt{5}}{10}$</p> <p>✓ area ΔEDM</p> <p>✓ correct method/ <i>korrekte metode</i></p> <p>✓ answer/antw</p> <p style="text-align: right;">(6) [19]</p>
---	--



QUESTION/VRAAG 5



<p>5.1</p>	$\sin \hat{C}AP = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$ <p>OR/OF</p> $\frac{\sin 90^\circ}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$	<p>✓ correct sine ratio/ korrekte sin-verh</p> <p>✓ $\frac{\sqrt{3}}{2}$</p> <p>(2)</p> <p>✓ correct sine ratio/ korrekte sin-verh</p> <p>✓ $\frac{\sqrt{3}}{2}$</p> <p>(2)</p>
<p>5.2</p>	<p>$\hat{C}PA = \hat{D}PA = 30^\circ$ (AP bisects $\hat{D}PC$)</p> $AD^2 = AP^2 + DP^2 - 2 \cdot AP \cdot DP \cdot \cos \hat{A}PD$ $= 8^2 + 4^2 - 2(8)(4) \cos 30^\circ$ $= 8^2 + 4^2 - 2(8)(4) \left(\frac{\sqrt{3}}{2}\right)$ $= 24,57\dots$ <p>AD = 4,96</p>	<p>✓ $\hat{D}PA = 30^\circ$</p> <p>✓ correct subst into cosine rule/ korrekte subst in cos-reël</p> <p>✓ 24,57...</p> <p>✓ 4,96</p> <p>(4)</p>

5.3	$\frac{\sin \hat{D} \hat{A} \hat{P}}{DP} = \frac{\sin \hat{A} \hat{P} \hat{D}}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30^\circ}{4,96}$ $\sin y = \frac{4 \sin 30^\circ}{4,96}$ $= 0,403\dots$ $y = 23,78^\circ$ <p style="text-align: center;">OR/OF</p> $AD^2 = AP^2 + DP^2 - 2 \cdot AP \cdot DP \cdot \cos \hat{D} \hat{A} \hat{P}$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96) \cdot \cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148\dots$ $y = 23,82^\circ$	<p>✓ correct subst into sine rule/ <i>korrekte subst in sin-reël</i></p> <p>✓ sin y subject</p> <p>✓ 23,78°</p> <p style="text-align: right;">(3)</p> <p>✓ correct subst into cosine rule/ <i>korrekte subst in cos-reël</i></p> <p>✓ cos y subject</p> <p>✓ 23,82°</p> <p style="text-align: right;">(3) [9]</p>
-----	---	--

QUESTION/VRAAG 6

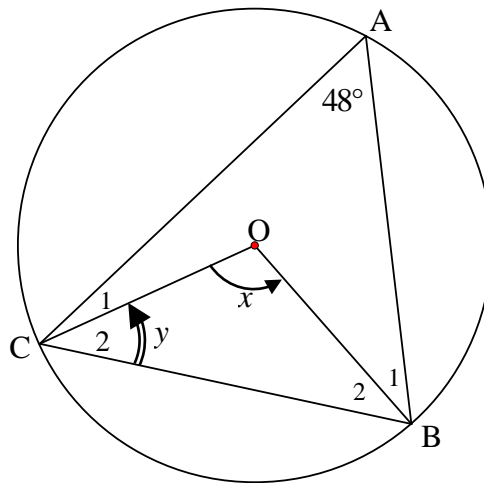
<p>6.1</p>	$\begin{aligned} & \cos^2(180^\circ + x) + \tan(x - 180^\circ) \sin(720^\circ - x) \cos x \\ & = (-\cos x)^2 + [-(-\tan x)] (-\sin x)(\cos x) \\ & = \cos^2 x + \left(\frac{\sin x}{\cos x} \right) (-\sin x)(\cos x) \\ & = \cos^2 x - \sin^2 x \\ & = \cos 2x \end{aligned}$	<p>✓ $(-\cos x)^2$ or $\cos^2 x$ ✓ $\tan x$ or $-(-\tan x)$ ✓ $-\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\cos^2 x - \sin^2 x$ (5)</p>
<p>6.2</p>	$\begin{aligned} & \sin(\alpha - \beta) \\ & = \cos[90^\circ - (\alpha - \beta)] \\ & = \cos[(90^\circ - \alpha) + \beta] \\ & = \cos(90^\circ - \alpha) \cos \beta - \sin(90^\circ - \alpha) \sin \beta \\ & = \sin \alpha \cos \beta - \cos \alpha \sin \beta \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & \sin(\alpha - \beta) \\ & = \cos[90^\circ - (\alpha - \beta)] \\ & = \cos[(90^\circ + \beta) + (-\alpha)] \\ & = \cos(90^\circ + \beta) \cos(-\alpha) - \sin(90^\circ + \beta) \sin(-\alpha) \\ & = (-\sin \beta) \cos \alpha - \cos \beta (-\sin \alpha) \\ & = \sin \alpha \cos \beta - \cos \alpha \sin \beta \end{aligned}$	<p>✓ rewrite as/herskryf $\cos[(90^\circ - \alpha) + \beta]$ ✓ expansion/ <i>uitbreiding</i> ✓ simpl/vereenv (3)</p> <p>✓ rewrite as/herskryf $\cos[(90^\circ + \beta) + (-\alpha)]$ ✓ expansion/ <i>uitbreiding</i> ✓ simpl/vereenv (3)</p>
<p>6.3</p>	$\begin{aligned} & x^2 - y^2 \\ & = \sin^2 76^\circ - \cos^2 76^\circ \\ & = -(\cos^2 76^\circ - \sin^2 76^\circ) \\ & = -\cos 2(76^\circ) \\ & = -\cos 152^\circ \\ & = -(-\cos 28^\circ) \qquad \qquad \qquad \mathbf{OR/OF} \qquad = -\cos(90^\circ + 62^\circ) \\ & = \cos 28^\circ \qquad \qquad \qquad = -(-\sin 62^\circ) \\ & = \cos(90^\circ - 62^\circ) \qquad \qquad \qquad = \sin 62^\circ \\ & = \sin 62^\circ \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & x^2 - y^2 \\ & = \sin^2 76^\circ - \cos^2 76^\circ \\ & = \sin 76^\circ \sin 76^\circ - \cos 76^\circ \cos 76^\circ \\ & = \sin 76^\circ \cos 14^\circ - \cos 76^\circ \sin 14^\circ \\ & = \sin(76^\circ - 14^\circ) \\ & = \sin 62^\circ \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & x^2 - y^2 \\ & = \sin^2 76^\circ - \cos^2 76^\circ \\ & = \cos^2 14^\circ - \sin^2 14^\circ \\ & = \cos 2(14^\circ) \\ & = \cos 28^\circ \\ & = \sin 62^\circ \end{aligned}$	<p>✓ $-(\cos^2 76^\circ - \sin^2 76^\circ)$ ✓ recognition of cos double angle ✓ $-\cos 152^\circ$ ✓ $\cos 28^\circ$ (4)</p> <p>✓ $\cos 14^\circ$ ✓ $\sin 14^\circ$ ✓ recognition of sine compound angle ✓ $\sin(76^\circ - 14^\circ)$ (4)</p> <p>✓ $\cos^2 14^\circ$ ✓ $\sin^2 14^\circ$ ✓ recognition of cos double angle ✓ $\cos 28^\circ$ (4)</p> <p style="text-align: right;">[12]</p>

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ kritieke waardes ✓ notation/notasie (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/st vorm (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = 0^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ$ or $x = 180^\circ + k \cdot 360^\circ$ or $x = 330^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ OR/OF $x = k \cdot 180^\circ, k \in \mathbb{Z}$	✓ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ ✓ $0^\circ ; 180^\circ$ OR/OF $x = k \cdot 180^\circ$ ✓ $210^\circ ; 330^\circ$ ✓ $k \cdot 360^\circ, k \in \mathbb{Z}$ (4)
7.4		✓ y-intercept/afsnit ✓ x-intercepts/afsnitte ✓ min/max points/ min/maks punte (3)
7.5	$f(x) = g(x)$ at/by: $x = -30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ $\therefore f(x + 30^\circ) = g(x + 30^\circ)$ at/by: $x = -60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$	✓ $-30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ ✓✓ $-60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$ (3)
7.6	Series will converge if/Reeks sal konvergeer as: $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore 30^\circ < x < 60^\circ$ or $x \in (30^\circ ; 60^\circ)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓✓ $30^\circ < x < 60^\circ$ (5) [19]

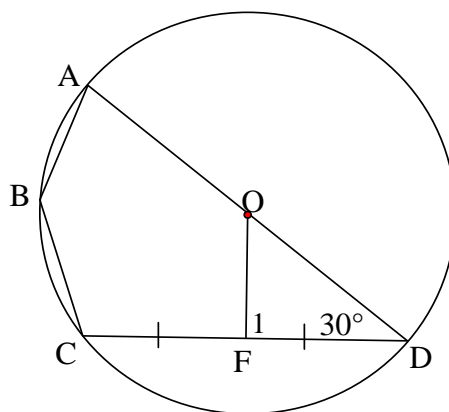
QUESTION/VRAAG 8

8.1



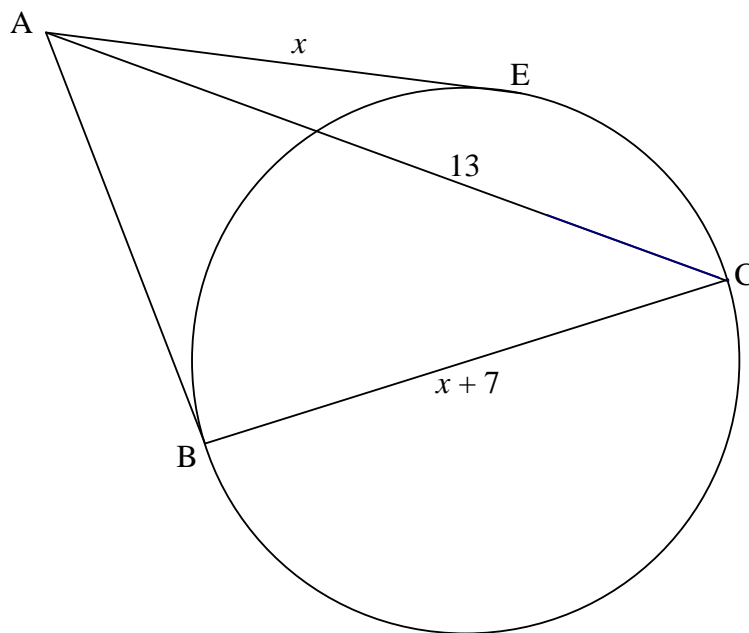
8.1.1	$x = 96^\circ$	(\angle at centre = $2\angle$ at circumference/ \angle by midpt = $2\angle$ by omtrek)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180^\circ - 96^\circ = 84^\circ$ $y = \hat{B}_2 = 42^\circ$	(sum of \angle s in Δ / som v \angle e in Δ) (\angle s opp = sides/ \angle e teenoor = sye)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90^\circ$	(line from centre to midpt chord/ lyn vanaf midpt na midpt kd)	✓ S ✓ R (2)
8.2.2	$\hat{A}\hat{B}\hat{C} = 150^\circ$	(opposite \angle s of cyclic quad/ tos \angle e v koordevh)	✓ S ✓ R (2)

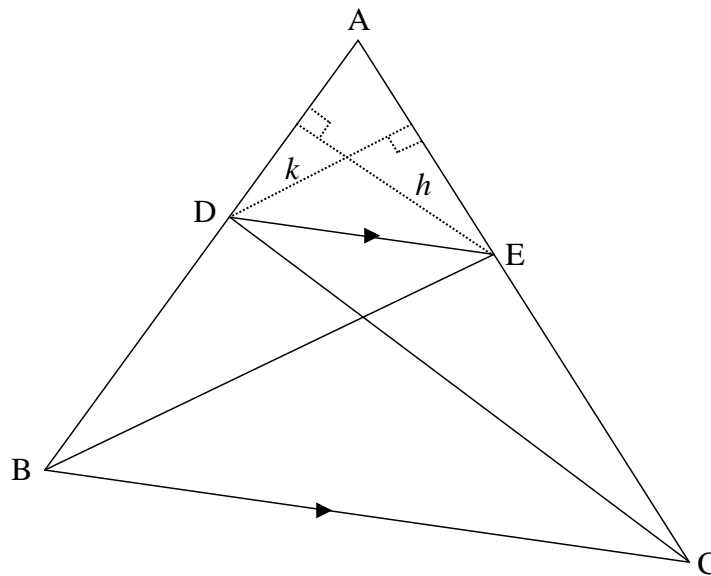
8.3



8.3.1 (a)	tangent \perp radius/diameter / <i>raaklyn \perp radius/middellyn</i>	✓ R (1)
8.3.1 (b)	tangents from common pt OR tangents from same pt / <i>raaklyne v gemeensk pt OF raaklyne vanaf dies pt</i>	✓ R (1)
8.3.2	$AB^2 + BC^2 = AC^2$ $x^2 + (x + 7)^2 = 13^2 \quad (\text{Theorem of/Stelling van Pythagoras})$ $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x - 5)(x + 12) = 0$ $x = 5 \quad (x \neq -12)$	✓ $AB^2 + BC^2 = AC^2$ ✓ $x^2 + (x + 7)^2 = 13^2$ ✓ standard form ✓ answer (4) [14]

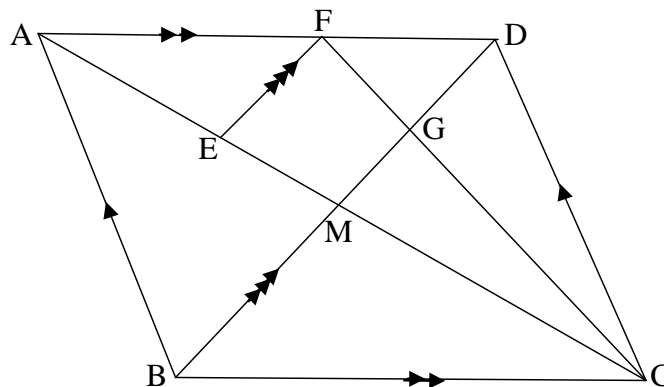
QUESTION/VRAAG 9

9.1



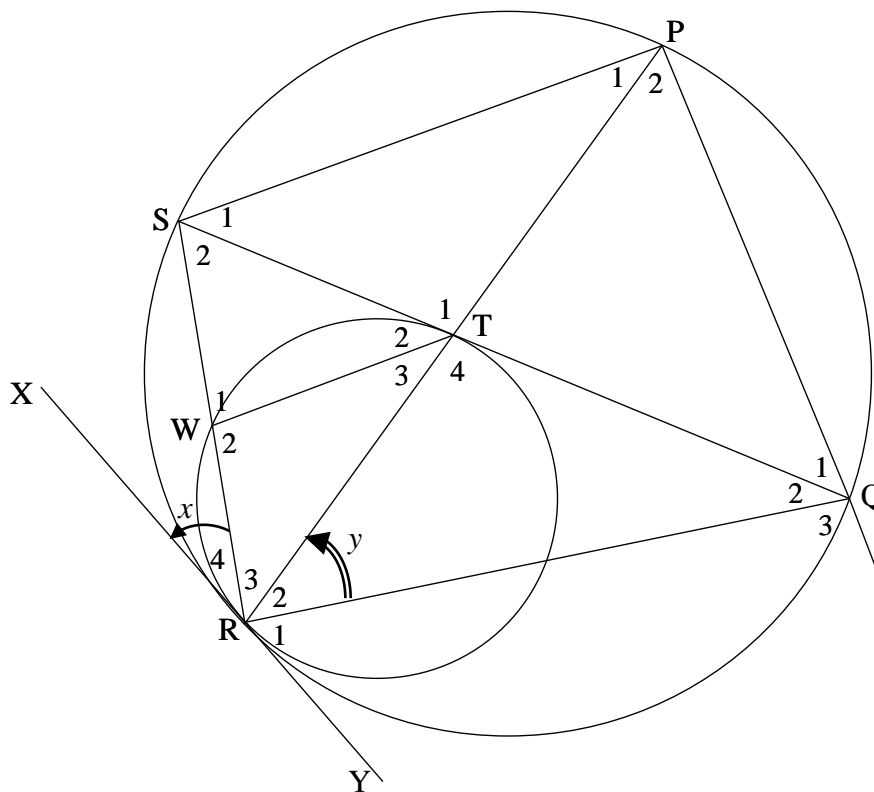
9.1.1	Same base (DE) and same height (between parallel lines) <i>Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)</i>	✓ same base/ <i>dies basis</i> between lines/ <i>tussen // lyne</i> (1)
9.1.2	$\frac{AD}{DB}$ $\frac{\frac{1}{2} AE \times k}{\frac{1}{2} EC \times k}$ <p>But/<i>Maar</i> area $\triangle DEB = \text{area } \triangle DEC$ (Same base and same height/<i>dieselfde basis en dieselfde hoogte</i>) $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$</p>	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

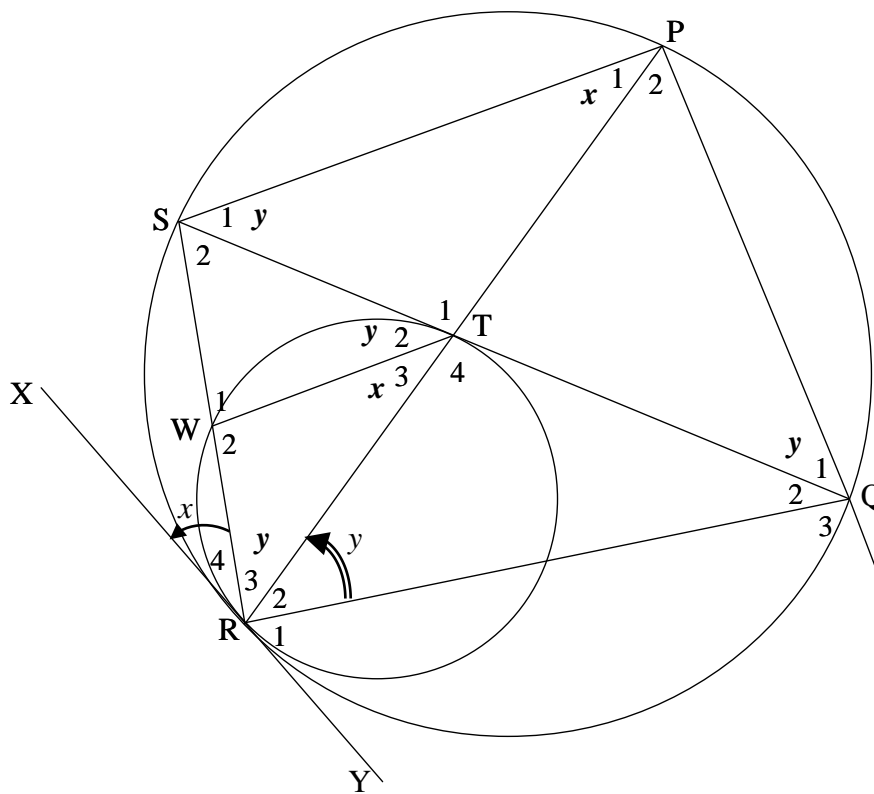


<p>9.2.1</p>	$\frac{EM}{AM} = \frac{FD}{AD}$ <p>(Line parallel one side of Δ OR prop th; $EF \parallel BD$) (<i>Lyn ewewydig aan sy v Δ</i> OF eweredigst; $EF \parallel BD$)</p> $\frac{EM}{AM} = \frac{3}{7}$	<p>✓ S ✓R</p> <p>✓ answer/antw</p> <p>(3)</p>
<p>9.2.2</p>	<p>$CM = AM$ $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$</p> <p>(diags of parm bisect/<i>hoekl parm halv</i>) (from 9.2.1/<i>vanaf 9.2.1</i>)</p>	<p>✓ S ✓R</p> <p>✓ answer/antw</p> <p>(3)</p>
<p>9.2.3</p>	<p>h of $\Delta FDC = h$ of ΔBDC ($AD \parallel BC$)</p> $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{\frac{1}{2} FD \cdot h}{\frac{1}{2} BC \cdot h}$ $= \frac{FD}{AD}$ <p>(opp sides of parm =) (<i>tos sye v parm =</i>)</p> $= \frac{3}{7}$ <p>OR/OF</p> $\frac{\text{area } \Delta FDC}{\text{area } \Delta ADC} = \frac{FD}{AD} = \frac{3}{7}$ <p>(same heights) (<i>dieselde hoogtes</i>)</p> <p>But Area $\Delta ADC =$ Area ΔBDC (diags of parm bisect area) (<i>hoekl v parm halv opp</i>)</p> $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{3}{7}$	<p>✓ $AD \parallel BC$</p> <p>✓ subst into area form/<i>subst in opp formule</i></p> <p>✓ S</p> <p>✓ answer/antw</p> <p>(4)</p> <p>✓ S ✓R</p> <p>✓ S</p> <p>✓ answer/antw</p> <p>(4)</p> <p>[16]</p>

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/Raaklyn-koordstelling	✓ R	(1)
10.1.2	Tangent chord theorem/Raaklyn-koordstelling	✓ R	(1)
10.1.3	Corresponding angles equal/Ooreenkomstige \angle e gelyk	✓ R	(1)
10.1.4	\angle s subtended by chord PQ OR \angle s in same segment <i>\anglee onderspan deur dieselfde koord OF \anglee in dieselfde segment</i>	✓ R	(1)
10.1.5	alternate \angle s/verwisselende \angle e ; WT SP	✓ R	(1)
10.2	$\frac{RW}{RS} = \frac{RT}{RP}$ <p>(Line parallel one side of Δ OR prop th; WT SP) (Lyn ewewydig aan sy v Δ OF eweredighst: WT SP)</p> <p>OR/OF</p> $\Delta RTW \parallel \Delta RPS$ <p>(\angle; \angle; \angle) ($\Delta RTW \parallel \Delta RPS$)</p> $\therefore \frac{RW}{RS} = \frac{RT}{RP}$ $\therefore RT = \frac{RW \cdot RP}{RS}$	✓ S ✓ R	(2)
10.3	$y = \hat{T}_2 = \hat{R}_3$ $y = \hat{R}_3 = \hat{Q}_1$ <p>(tan chord theorem/Rkl-koordst) (\angles in same segment/\anglee in dieselfde segment)</p>	✓ S ✓ R ✓ S ✓ R	(4)



<p>10.4</p>	<p>$\hat{Q}_3 = \hat{P}SR$ (ext \angle of cyc quad/buite \angle v kdvh) $\hat{P}SR = \hat{W}_2$ (corresp \angles/ooreenk \anglee ; WT SP) $\therefore \hat{Q}_3 = \hat{W}_2$ OR/OF $\hat{Q}_2 = x$ (\angles in same segment/\anglee in dies segment) $\hat{Q}_3 = 180^\circ - (x + y)$ (\angles on straight line/\anglee op reguitlyn) $\hat{W}_2 = 180^\circ - (x + y)$ (\angles of ΔWRT/\anglee v ΔWRT) $\therefore \hat{Q}_3 = \hat{W}_2$</p>	<p>✓ S ✓ R ✓ S ✓ R ✓ S ✓ S (3)</p>
<p>10.5</p>	<p>In ΔRTS and ΔRQP: $\hat{R}_3 = \hat{R}_2 = y$ (proven above/hierbo bewys) $\hat{S}_2 = \hat{P}_2$ (\angles in same segment/\anglee in dies segment) $\hat{R}TS = \hat{R}QP$ (3rd angle of Δ) $\therefore \Delta RTS \parallel \Delta RQP$ (\angle; \angle; \angle)</p>	<p>✓ S ✓ S/R ✓ S OR/OF (\angle; \angle; \angle) (3)</p>

<p>10.6</p>	$\frac{RT}{RQ} = \frac{RS}{RP} \quad (\triangle RTS \parallel \triangle RQP)$ $\frac{RS}{RP} \times \frac{RS}{RP} = \frac{RT}{RQ} \times \frac{RS}{RP}$ $\left(\frac{RS}{RP}\right)^2 = \left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ $= \left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right) \quad (\text{proven in 10.2/bewys in 10.2})$ $= \frac{RW}{RQ}$ <p>OR/OF</p> $\frac{RT}{RQ} = \frac{RS}{RP} \quad (\triangle RTS \parallel \triangle RQP)$ <p>But $RT = \frac{WR.RP}{RS}$ (proven in 10.2/bewys in 10.2)</p> $\therefore \frac{RT}{RQ} = \frac{WR.RP}{RQ.RS} = \frac{RS}{RP}$ $WR.RP^2 = RQ.RS^2$ $\therefore \frac{WR}{RQ} = \frac{RS^2}{RP^2}$ <p>OR/OF</p> $\frac{RT}{RS} = \frac{RQ}{RP} \quad (\triangle RTS \parallel \triangle RQP)$ $RQ = \frac{RT.RP}{RS}$ <p>and $WR = \frac{RT.RS}{RP}$ (proven in 10.2/bewys in 10.2)</p> $\frac{WR}{RQ} = \frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$ $= \frac{RT.RS}{RP} \times \frac{RS}{RT.RP}$ $= \frac{RS^2}{RP^2}$	<p>✓ S</p> <p>✓ $\times \frac{RS}{RP}$ on both sides</p> <p>✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ (3)</p> <p>✓ S</p> <p>✓ $RT = \frac{WR.RP}{RS}$</p> <p>✓ multiplication/ vermenigvuldig (3)</p> <p>✓ S</p> <p>✓ $WR = \frac{RT.RS}{RP}$</p> <p>✓ simplification/ vereenvoudiging (3)</p> <p>[20]</p>
<p>TOTAL/TOTAAL: 150</p>		