



**MATHEMATICS: PAPER II**

EXAMINATION NUMBER

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Time: 3 hours

150 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number on the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
8. It is in your own interest to write legibly and to present your work neatly.
9. Round off to two decimal places unless otherwise stated.

**FOR OFFICE USE ONLY: MARKER TO ENTER MARKS**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
7	14	18	11	12	15	12	14	8	12	12	15	/150

**SECTION A**

**QUESTION 1**

A study is done with twelve employees in a company to understand the relationship between the number of rest days given in a year and the productivity of each employee.

The results are shown in the table below:

Rest days given	5	2	9	1	3	12	10	4	4	5	8	6
Productivity of the employee	0,87	0,65	0,9	0,58	0,7	0,91	0,88	0,78	0,72	0,91	0,82	0,62

- (a) Calculate the correlation coefficient. (Round off correct to four decimal places.)

\_\_\_\_\_ (2)

- (b) Refer to your correlation coefficient and circle the letter that best describes the relationship. Only one letter must be circled.

A Fairly strong, negative correlation

B Very weak, positive correlation

C Fairly strong, positive correlation

D Perfect, positive correlation

(1)

- (c) If the equation of the least squares regression line is  $y = A + Bx$ , calculate the values of A and B. Give answers correct to four decimal places.

\_\_\_\_\_  
 \_\_\_\_\_ (2)

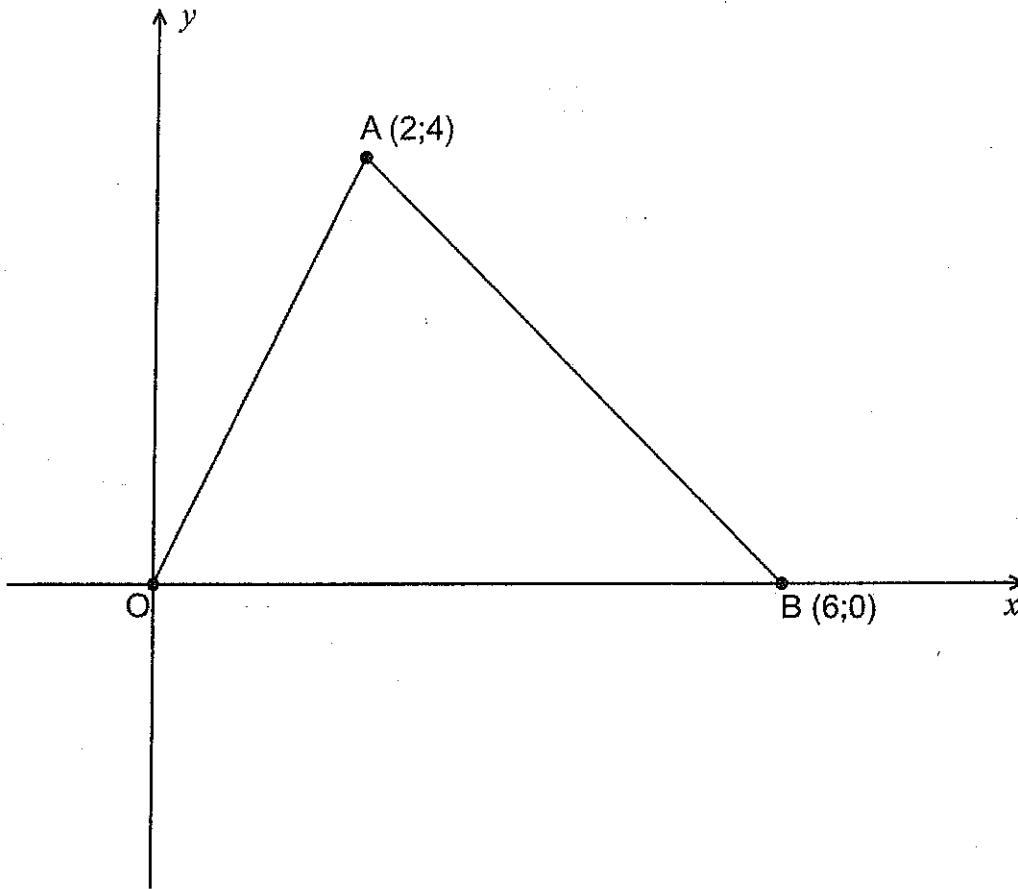
- (d) Should the regression line in (c) be used to predict the productivity of an employee if thirty rest days were given to the employee in a year? (Explain your answer.)

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(2)  
 [7]

**QUESTION 2**

In the Cartesian plane below,  $\triangle OAB$  with  $O(0;0)$ ,  $A(2;4)$  and  $B(6;0)$  is drawn.



- (a) Calculate the gradient of OA and hence the size of  $\hat{A}OB$ .

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(4)

(b) Determine the equation of the perpendicular bisector of OA.

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(4)

(c) Write down the equation of the perpendicular bisector of OB.

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(1)

(d) Determine the equation of the circle passing through O, A and B.

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(5)  
[14]

**QUESTION 3**

(a) If  $\sin 31^\circ \cdot \cos 22^\circ + \sin 22^\circ \cdot \cos 31^\circ = k$ , then without the use of a calculator, determine the value of the following in terms  $k$ :

(1)  $\sin 53^\circ$

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(1)

(2)  $\cos 143^\circ$

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(2)

(3)  $\sin 75^\circ \cdot \sin 22^\circ + \cos 75^\circ \cdot \cos 22^\circ$

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(3)



- (c) Calculate the general solution for  $\theta$  if  $3 \sin^2 \theta = 2 \sin \theta$ . Give answer correct to one decimal place.

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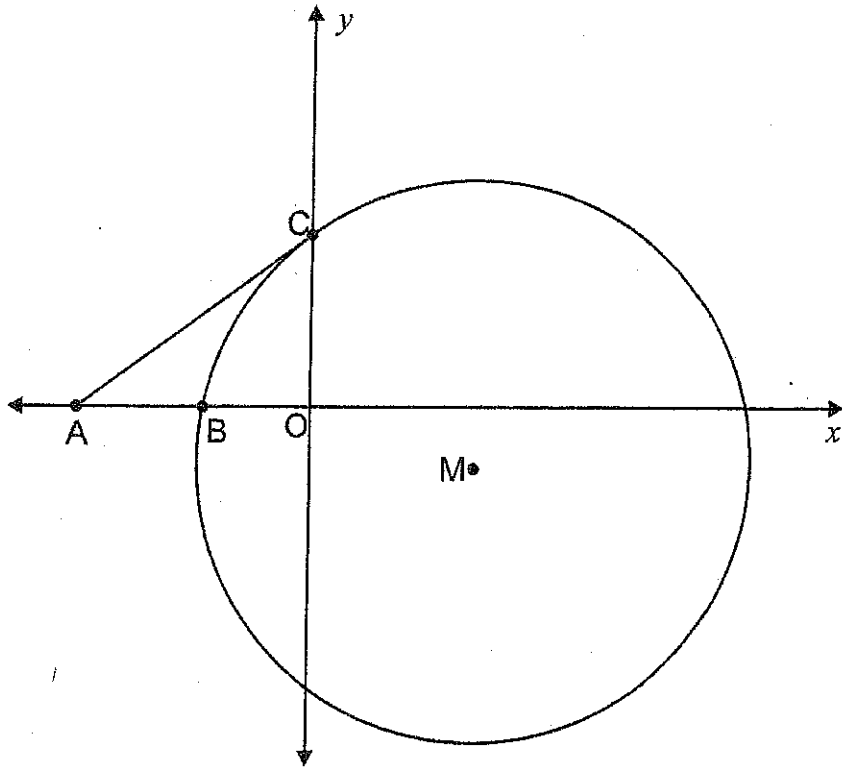
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(6)  
[18]

**QUESTION 4**

In the Cartesian plane below, circle centre M is drawn.

- A is a point on the x-axis.
- Point B lies on the circle and the x-axis.
- Point C lies on the circle and the y-axis.
- The equation of the circle is  $(x - 3)^2 + (y + 1)^2 = 25$ .
- Line AC is a tangent to the circle at C.



(a) Write down the coordinates of M.

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(1)

(b) Determine the coordinates of point C.

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(3)



(c) Determine the equation of the tangent AC.

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(3)

(d) Determine the length of AB. Leave your answer correct to one decimal place.

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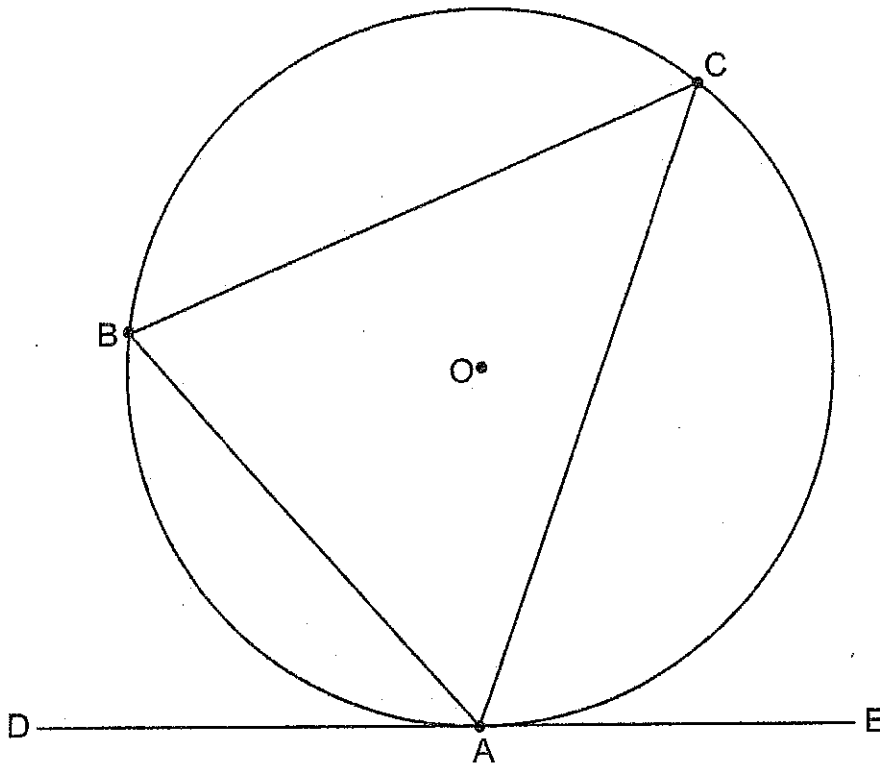
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(4)  
**[11]**

**QUESTION 5**

- (a) Prove the theorem that states the angle between a tangent and a chord is equal to the angle in the alternate segment.



Given: DE is a tangent to circle centre O at A.  
B and C are points on the circle.

Required to prove: \_\_\_\_\_ (1)

Construction: \_\_\_\_\_ (1)

Proof:

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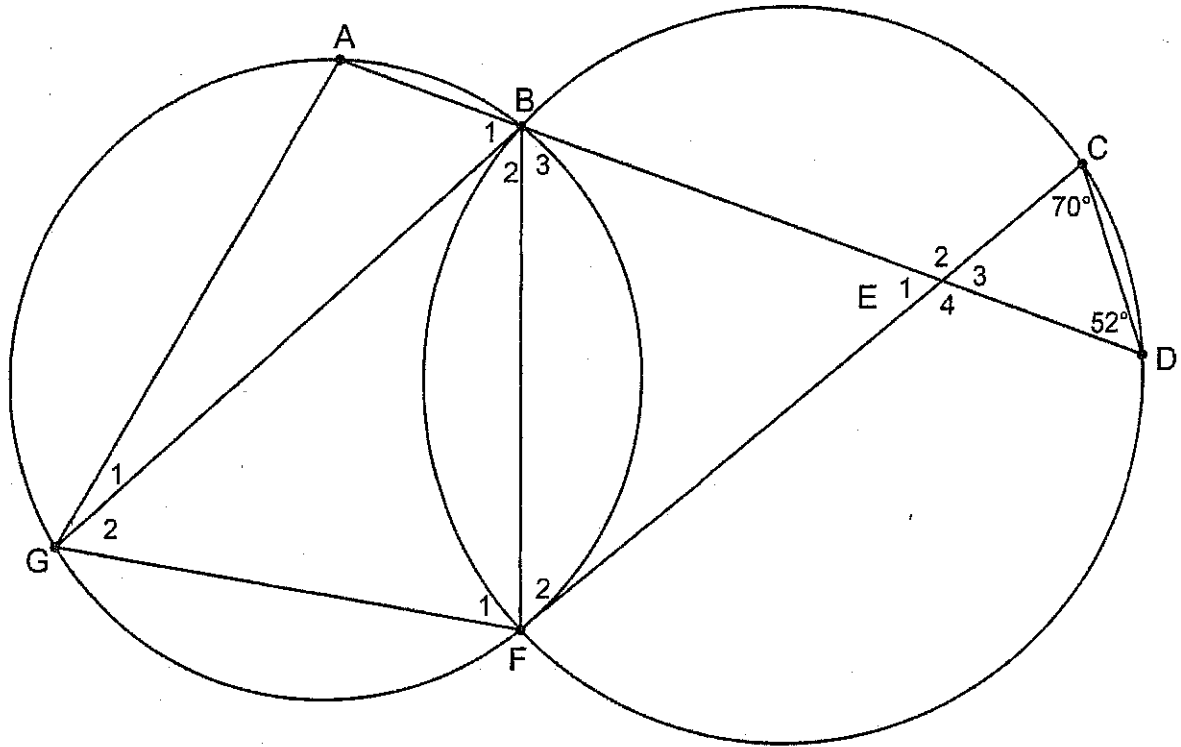
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(5)

(b) In the diagram below, two circles are drawn intersecting at B and F.

- CF is a tangent to the smaller circle at F.
- A and G are points on the circumference of the smaller circle.
- Chords FC and BD of the larger circle intersect at E.
- ABD is a straight line.
- $\hat{C} = 70^\circ$  and  $\hat{D} = 52^\circ$ .



Determine the size of  $\hat{G}_1$ .

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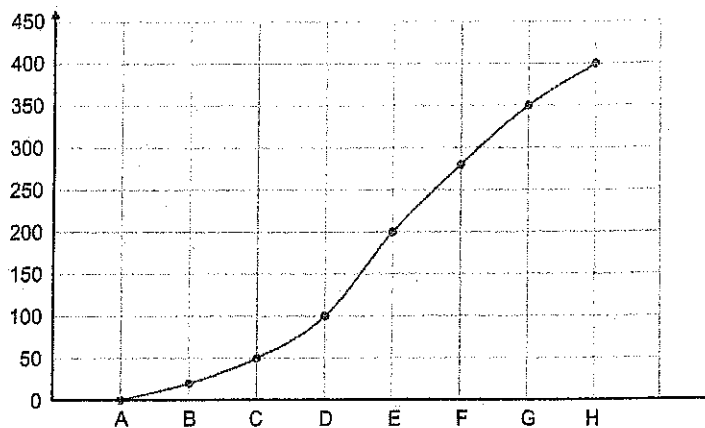
(5)  
[12]

**QUESTION 6**

A number of learners were asked how many WhatsApp messages they sent during a day.

The results are summarised in the table and the cumulative frequency curve given below.

WhatsApp messages sent	Frequency
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	P
$200 \leq x < 250$	M
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50



- (a) What is the value of A on the horizontal axis of the cumulative frequency curve?

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(1)

- (b) How many learners were asked for information?

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(1)

- (c) Determine the values of P and M in the table above.

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(2)

- (d) Calculate the interquartile range.

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(3)

- (e) Calculate an estimate for the mean WhatsApp messages sent per day.

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(2)

(f) If the cellphone company introduced a contract where you were not allowed to send more than 300 WhatsApp messages per day then:

(1) How would this affect the median? Explain.

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(2)

(2) How would this affect the standard deviation? Explain.

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(2)

(3) In which direction would the data be skewed? Explain.

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(2)  
[15]

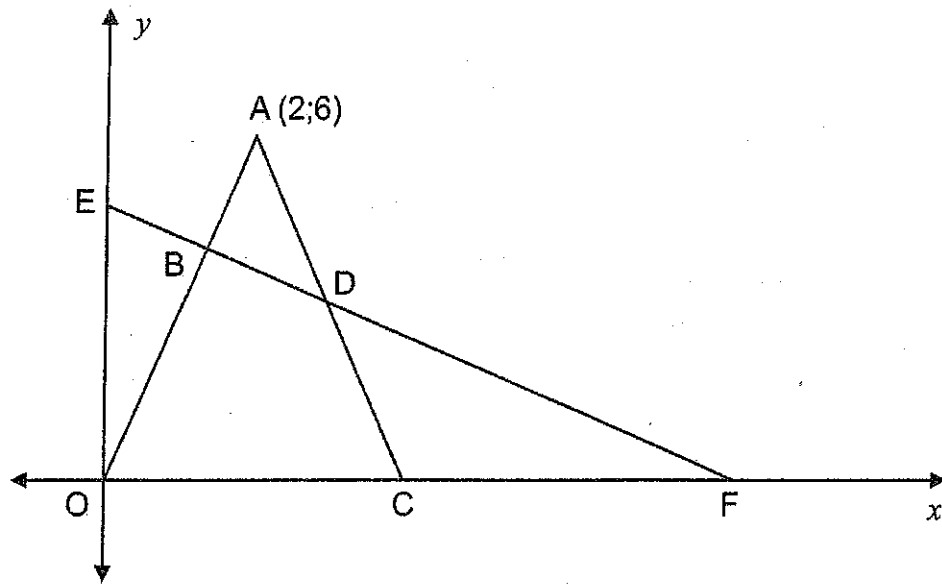
**77 marks**

**SECTION B**

**QUESTION 7**

In the diagram below,  $\triangle AOC$  with  $A(2;6)$  and  $O(0;0)$  is drawn.

- C is a point on the x-axis.
- $AO = AC$ .
- E lies on the y-axis and F lies on the x-axis.
- Line EF goes through the points B and D on OA and CA respectively.
- The equation of EF is given by  $2y + x = 10$ .



(a) Determine the coordinates of B and hence the area of  $\triangle EBO$ .

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(8)

- (b) If  $D\left(x; \frac{18}{5}\right)$ , determine the area of  $\Delta DCF$ .

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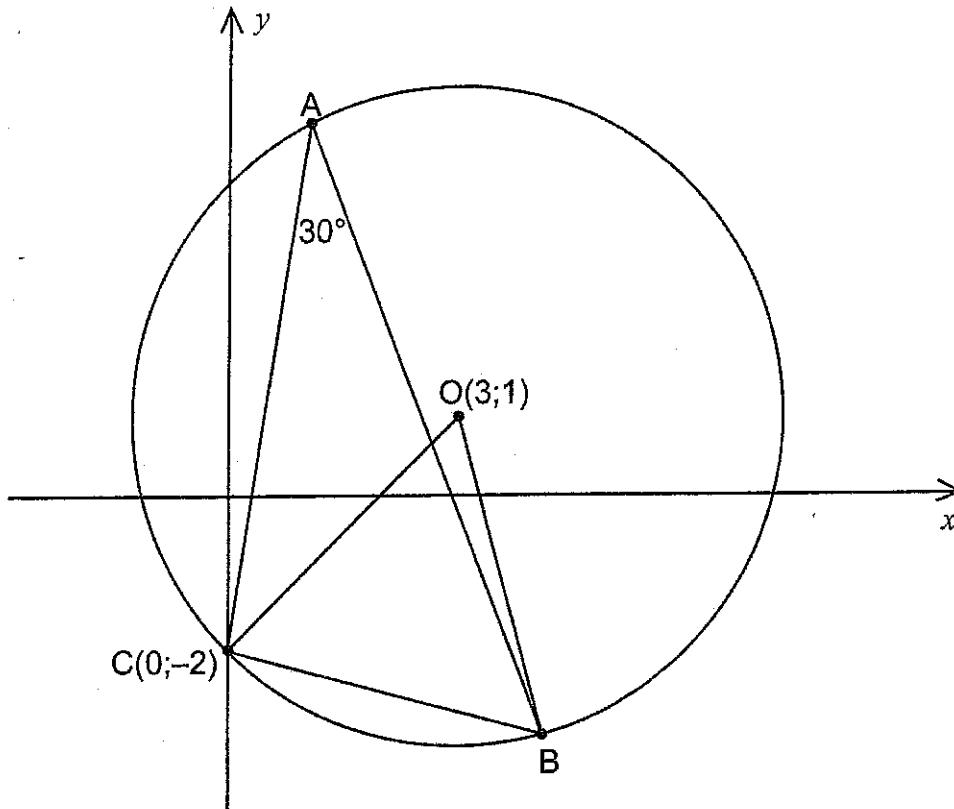
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(4)  
[12]

**QUESTION 8**

In the Cartesian plane below, circle centre  $O(3;1)$  is drawn.

- A and  $C(0;-2)$  are fixed points on the circle.
- $\hat{CAB} = 30^\circ$ .
- B is a variable point on the circle.



- (a) (1) Determine the length of OC.

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(2)

- (2) B moves along the circle until BC is parallel to the x-axis. Write down the new coordinates of B.

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(2)



- (3) Calculate the size of  $\hat{CAB}$  for this new position of B. Give all reasons.

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(4)

- (b) B moves from its **original position** along the circle in an anti-clockwise direction until the area  $\Delta OBC = \frac{9}{2}$  square units.  
 Find the shortest distance that B has to move along the circle for the above to occur.

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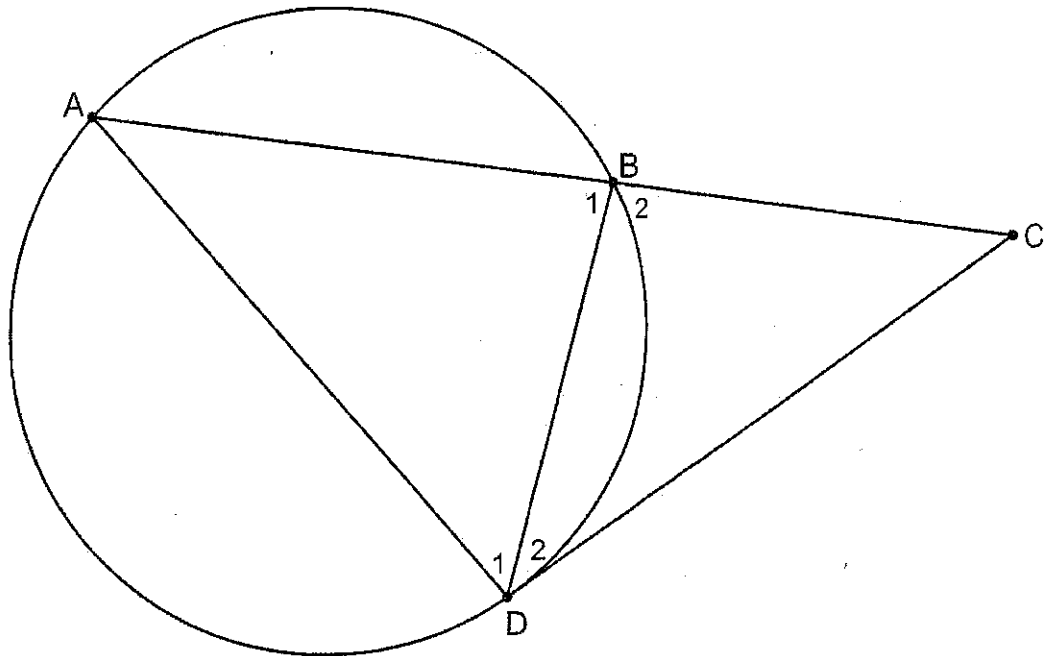
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(6)  
 [14]

**QUESTION 9**

In the diagram below, a circle passing through A, B and D is drawn.

- CD is a tangent to the circle at D.



- (a) Prove that  $\triangle ADC \sim \triangle DBC$ .

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(4)

- (b) Show that  $AB \cdot BC = DC^2 - BC^2$ .

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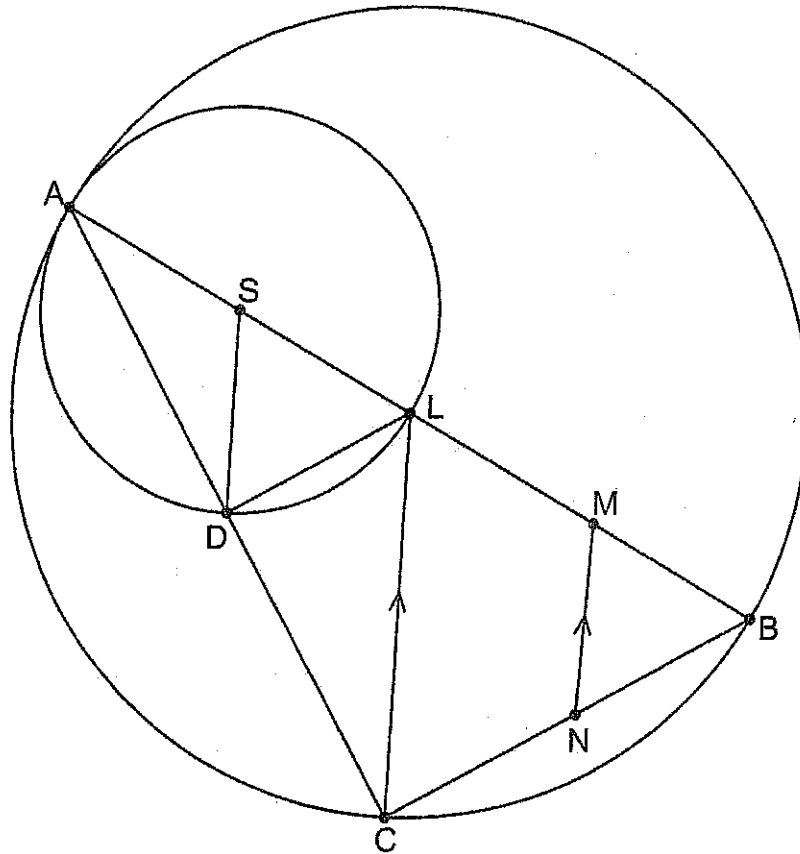
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(4)  
[8]

**QUESTION 10**

In the diagram below, two circles touch internally at A.

- AB is the diameter of the larger circle and AL is the diameter of the smaller circle.
- S and L are the centres of the circles.
- D is a point on the smaller circle and C is a point on the larger circle. ADC is a straight line.
- M is a point on LB so that  $MN \parallel LC$ .



(a) Prove that  $DL \parallel CB$ .

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(4)

(b) Prove that  $2SD = LC$ .

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(3)

(c) Determine the value of  $\frac{SL}{AB}$ .

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(2)

(d) If  $AB = 30$  units and  $\frac{BN}{NC} = \frac{7}{9}$ , then determine the length of  $LM$ .

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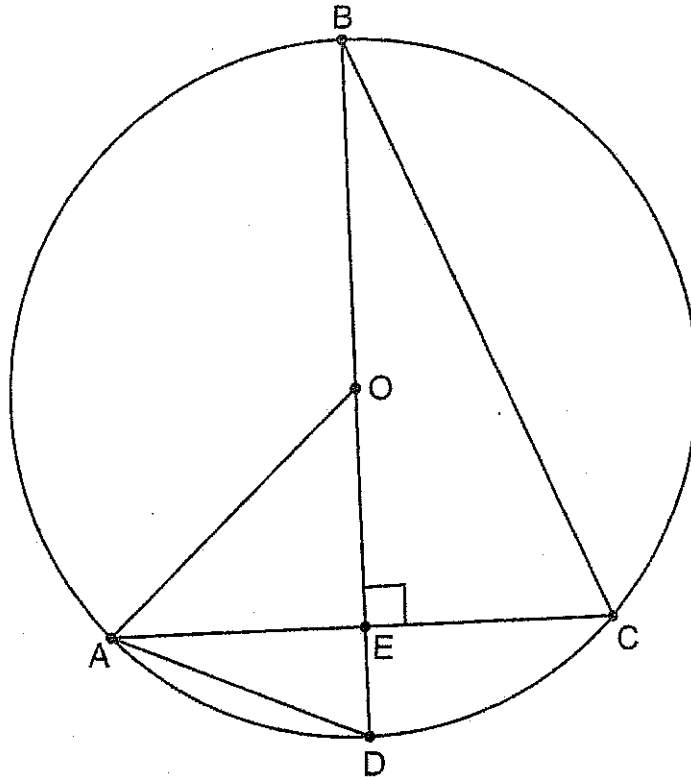
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(3)  
[12]

**QUESTION 11**

(a) In the diagram below, a circle with centre O is drawn.

- $OD \perp AC$  and OD and AC intersect at E.
- A, B, C and D lie on the circumference of the circle.



(1) Determine the length of BE in terms of AO and ED.

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(2)

(2) Prove that  $(2AO - ED)^2 = BC^2 - AE^2$ .

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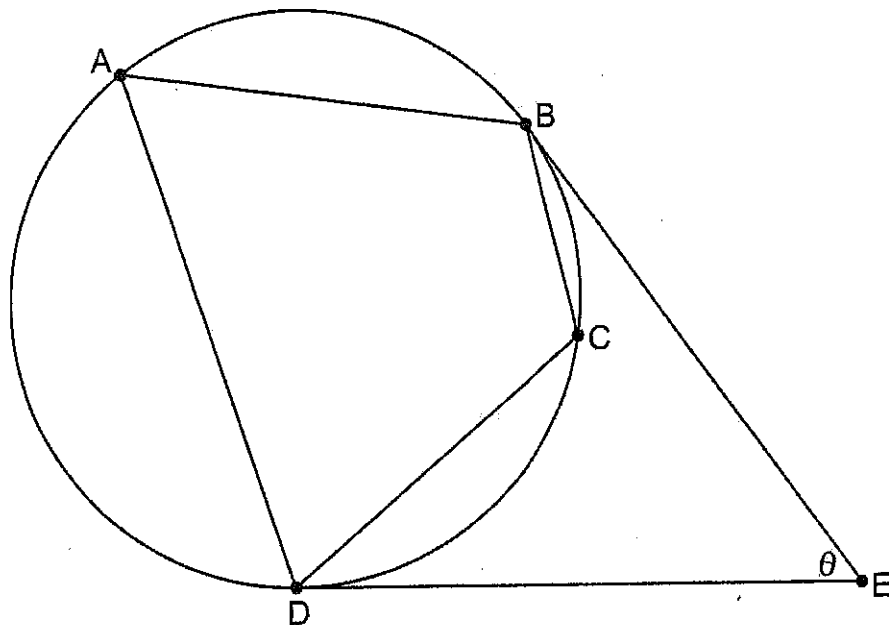


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(4)

(b) In the diagram below, a circle is drawn passing through A, B, C and D.

- $\hat{B}ED = \theta$ .
- BE and ED are tangents at B and D respectively.



Prove that  $\hat{B}CD = 90^\circ + \frac{\theta}{2}$ .

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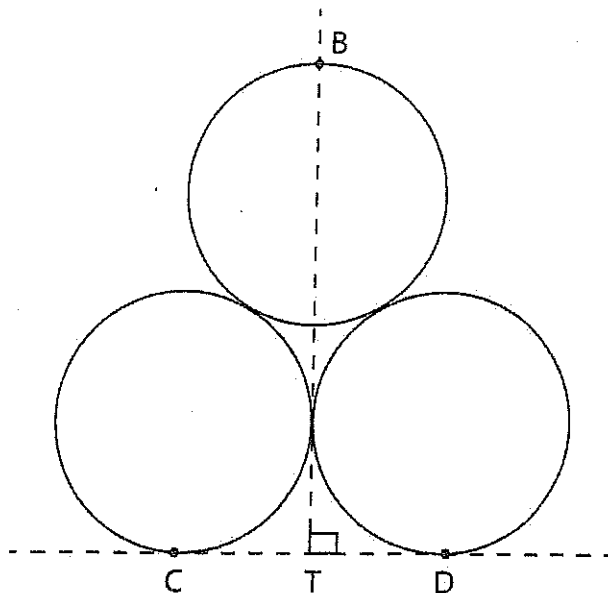


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(6)  
[12]

**QUESTION 12**

- (a) In the diagram below, three EQUAL circles of radius 3 units are positioned so that they touch each other. BT is a vertical common tangent to two circles and CD is a horizontal common tangent to the same circles.



Show that the length of  $BT = 3\sqrt{3} + 6$ .

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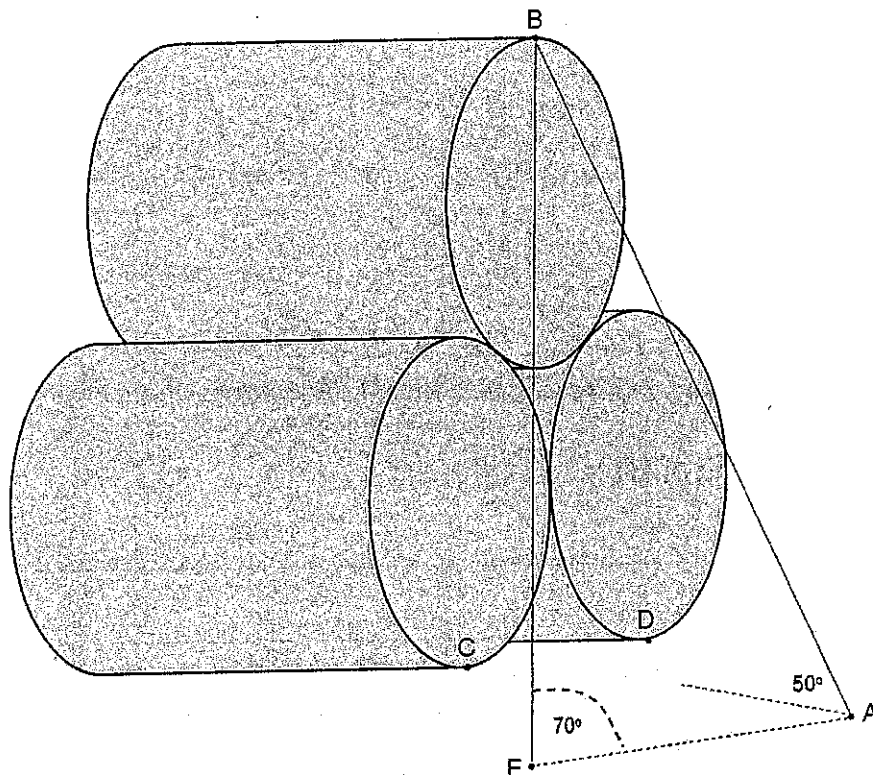


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(5)

(b) Three **identically sized** cylinders are stacked on top of each other as shown in the diagram below. They are anchored down by a piece of rope from A to B and another piece of rope from B to E.

- A, C, D and E lie on the same horizontal plane.
- B, C and D lie on the same vertical plane.
- B is the highest point on the cylinder.
- The angle of elevation from A to B is  $50^\circ$ .
- $\hat{BEA} = 70^\circ$ .
- The radius of each cylinder is 3 metres.



(1) Calculate the length of AB (the rope required to anchor the cylinder down).

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(4)



- (2) If the second rope EB has a length of 13 metres then determine the straight-line distance between E and A.

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(6)  
[15]

**73 marks**

**Total: 150 marks**