NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2016

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) Opp angles do not add up to 180°.

(b) $\tan 45° = m$
$m = 1$
$y = x + 8$

(c) (1) $x = 6$

(2) B(6 ; 14) $\therefore$ Area $= \frac{1}{2}(8+14)(6) = 66 \text{ units}^2$

QUESTION 2

(a) (1) $\text{M} = \dfrac{2\sin^2\theta + 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$\text{M} = \dfrac{2\sin\theta(\sin\theta + \cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$

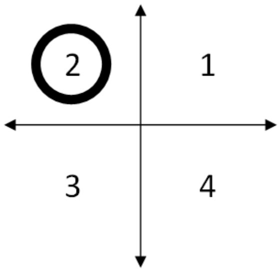
$\text{M} = \dfrac{2\sin\theta}{(\cos\theta - \sin\theta)}$

Therefore

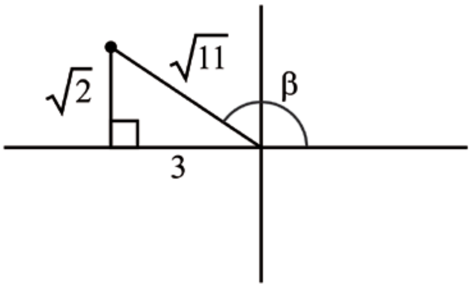
$\text{M} = \text{P}$

(2) $\cos\theta - \sin\theta = 0$
$\cos\theta = \sin\theta$
$1 = \tan\theta$
Reference angle: 45°
$\theta = \{-135°; 45°; 225°\}$

(b) (1)



(2) **Main**



$\tan\beta = \frac{y}{x} = \frac{\sqrt{2}}{-3}$

Alternate:

$\left(\sqrt{11}\right)^2 = \left(\sqrt{2}\right)^2 + x^2$

$x^2 = 9$

$x = \pm 3$

Since $\sin\beta > 0,$ and $\cos\beta < 0$ is in the second quadrant

Quadrant Two

$x = -3$ (This mark is for the accuracy of the sign.)

$y = \sqrt{2}$

$r = \sqrt{11}$

$= \tan\beta$

$= -\frac{\sqrt{2}}{3}$

(c) (1) $\cos(\alpha - 30°) - \cos(\alpha + 30°)$

$= \cos\alpha\cos 30° + \sin\alpha\sin 30° - (\cos\alpha\cos 30° - \sin\alpha\sin 30°)$

$= \cos\alpha\cos 30° + \sin\alpha\sin 30° - \cos\alpha\cos 30° + \sin\alpha\sin 30°$

$= 2\sin\alpha\sin 30° \quad = 2\sin\alpha \times \left(\frac{1}{2}\right)$

$= \sin\alpha$

(2) $\sin\alpha = 2\sin^2\alpha$

$0 = \sin\alpha(2\sin\alpha - 1)$

$\sin\alpha = 0$ OR $\sin\alpha = \frac{1}{2}$

$\alpha = 0° + k180°$

$\left.\begin{array}{l}\alpha = 30° + k360° \\ \alpha = 150° + k360°\end{array}\right\}$ $k \in \mathbb{Z}$

OR

$\left(\alpha = 0° + k360° \text{ or } \alpha = 180° + k.360°\right)$

QUESTION 3

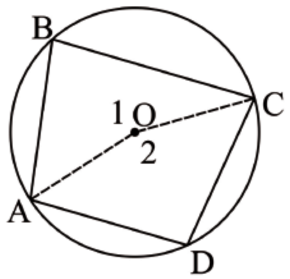
(a) Radius of circle Q is $9-5=4$ units. (If on diagram then they get the mark)
x_{Q} of the centre of the circle is $9+5=14$ units.
y_{Q} of the centre of the circle is 5 units.
therefore the equation of the circle is $(x-14)^2+(y-5)^2=16$

(b) $(x-p)^2+y^2-22y+121=-117+121$
$(x-p)^2+(y-11)^2=4$
Therefore RQ is 6 units. (Note: If they use 2 + 4 they can get a max of 2 out of 3)

(c) $\text{PR}=\sqrt{(11-5)^2+(14-5)^2}$
$\text{PR}=\sqrt{117}$
$\therefore \text{AB}=\sqrt{117}-2-5$
$=3{,}82$ units (Full marks for the correct answer)

QUESTION 4

(a) Draw AO and OC
R.T.P: $\hat{\text{B}}+\hat{\text{D}}=180°$
Proof:



$\hat{\text{O}}_2=2\times\hat{\text{B}}$ (Angle at centre)
$\hat{\text{O}}_1=2\times\hat{\text{D}}$ (Angle at centre)
$\hat{\text{O}}_1+\hat{\text{O}}_2=360°$
$\therefore 2\hat{\text{B}}+2\hat{\text{D}}=360°$
$\therefore \hat{\text{B}}+\hat{\text{D}}=180°$

(b) **Main:**
$\text{A}\hat{\text{B}}\text{C}=62°$ (tan chord theorem)
$\text{A}\hat{\text{O}}\text{C}=124°$ (Angle at centre is twice the angle at the circumference)
$\hat{\text{C}}_2=\hat{\text{A}}_3=28°$ (*isos*Δ **OR** OC = OA)
$\therefore \hat{\text{C}}_1=37°$ (Δ's in a Δ)
OR
Alternate:
$\hat{\text{A}}_3=90°-62°$ (radius $\perp$ tangent)
$=28°$
$\hat{\text{C}}_2=\hat{\text{A}}_3=28°$ (*isos*Δ **OR** OC = OA)
$\text{A}\hat{\text{B}}\text{C}=62°$ (tan chord theorem)
$\therefore \hat{\text{C}}_1=37°$ (Δ's in a Δ)

(c) (1) N = Q OR M+N = M + Q

(2) $\hat{D}_1 = \hat{B}$ (exterior angle of a cyclic quad)
$\hat{D}_1 = \hat{A}_1 + \hat{C}_2$ (exterior angle of triangle = sum of the two int opp angles)
$\therefore \hat{B} = \hat{A}_1 + \hat{C}_2$

QUESTION 5

(a) $y = 3\sin 360° + 1$
$y = 1$
$A(360°;1)$

(b) $3\sin x + 1 = -1$
$3\sin x = -2$
$\sin x = \frac{-2}{3}$
Reference Angle: $41{,}81°$
$x = \{221{,}81°;\ 318{,}19°\}$

(c) $k > 4$ **OR** $k < 1$

QUESTION 6

(a) $r = 0.9755$
Very strong

(b) $A = 2\ 788{,}26$
$B = 1\ 658{,}39$

Line of best fit.
$y = 1\ 658{,}39x + 2\ 788{,}26$

(c) Sub in 19 for *x*.
$y = 1\ 658{,}39(19) + 2\ 788{,}26$
$y = R34\ 297{,}67$

His projected income based on his line of best fit is R34 297,67.

The manager would not consider this a successful day.

OR
NO; Link to the table using the number 17; logical argument

SECTION B

QUESTION 7

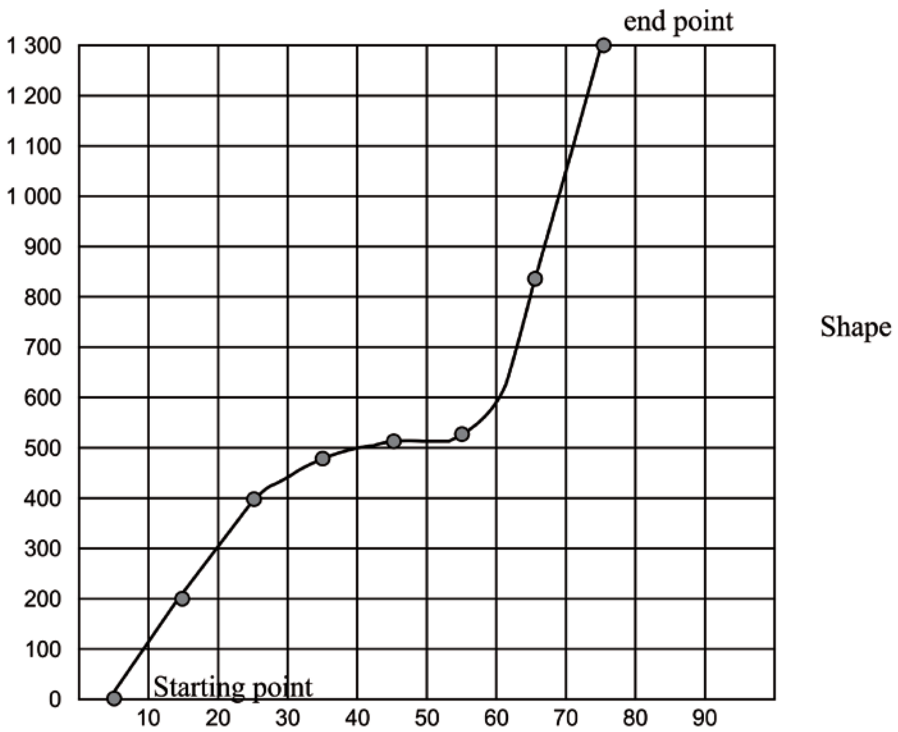
(a) $A = 250$
$B = 502$

(b) (1) $\bar{x} \approx \frac{200(10)+250(20)+20(30)+32(40)+23(50)+300(60)+475(70)}{1\,300}$

$\bar{x} \approx 47{,}14$ (If they get the right answer and no working is shown, then full marks.)

(2) $65 < x \leq 75$

(c)



(d) (1) No. Skewed to the left as the mean is less than the median.
OR
Bimodel, big dip in the middle.
OR
Shape of Ogive is not correct.

(2) No. It is not a good indicator as the majority of the people who use your product are between 65 and 75.
OR
Yes. It is a good indicator as the people in this age range will be looking to buy the product for their children between the ages of 5 and 25.
(Many answers to be considered.)

QUESTION 8

(a) TR =3
(TP $\perp$ OP or $OR \perp RT$ OR drawn on diagram)
$\mathrm{OP}^2 = 5^2 - 3^2$
OR = 4
$\mathrm{OP} = \mathrm{OR}$ (tangents drawn from same point)
$\therefore x_{\mathrm{T}} = 4$
$\mathrm{T}(4;3)$ (Workings can be shown on diagram)

(b) $\tan \mathrm{T\hat{O}R} = \frac{3}{4}$
$\mathrm{T\hat{O}R} = 36,87°$

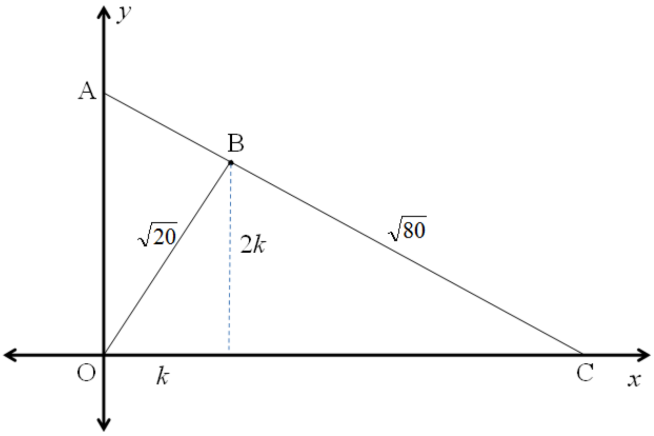
(c) $\mathrm{P\hat{O}R} = 2 \times 36,87° = 73,74°$ (properties of kite OPTR)
$\sin \mathrm{P\hat{O}R} = \frac{y_{\mathrm{P}}}{4}$
$y_{\mathrm{P}} = 3,84$ units

QUESTION 9

(a) $\mathrm{OC}^2 = 80 + 20$
$\mathrm{OC} = 10$ pythagoras

(b) **Main:**
$\tan \mathrm{B\hat{C}O} = \frac{\sqrt{20}}{\sqrt{80}}$
$= 26,57°$
$m_{\mathrm{AC}} = -\tan 26,57$
$m_{\mathrm{AC}} = -0,5$
OR
Alternate:
Gradient of line $\mathrm{AC} = -1 \times \frac{AO}{OC}$
$(\Delta \mathrm{ABO} /// \Delta \mathrm{OBC})$
$\frac{\mathrm{AO}}{\mathrm{OC}} = \frac{\mathrm{BO}}{\mathrm{BC}} = \frac{\sqrt{20}}{\sqrt{80}}$
Gradient of $\mathrm{AC} = -1 \times \frac{\sqrt{20}}{\sqrt{80}} = -\frac{1}{2}$
OR
Alternate:
$\tan \mathrm{O\hat{C}B} = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2}$
$\therefore m_{\mathrm{AC}} = \tan(180° - \mathrm{O\hat{C}B})$
$= -\tan \mathrm{B\hat{C}O}$
$= -\frac{1}{2}$

(c) Gradient of $\text{OB} = -\frac{1}{m_{\text{AC}}} = 2$



$(2k)^2 + k^2 = 20$ (coordinates of point B)

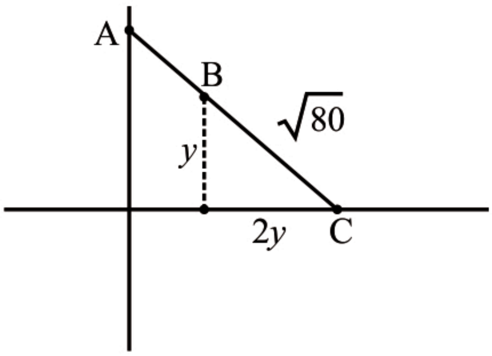
$5k^2 = 20$ $B(2;\ 4)$

$k = 2$

OR

Alternate

$\tan \text{O}\hat{\text{C}}\text{B} = \frac{1}{2}$



$y^2 + 4y^2 = 80$

$y = 4$

$x_{\text{B}} = 10 - 2(4)$

$x_{\text{B}} = 2$

$B(2;4)$

(d) Let $\text{C}\hat{\text{O}}\text{B} = \theta$

$\text{A}\hat{\text{O}}\text{B} = 90° - \theta$

$\therefore \text{O}\hat{\text{A}}\text{B} = 90° - (90° - \theta) = \theta$

$\therefore \text{O}\hat{\text{A}}\text{B} = \text{C}\hat{\text{O}}\text{B}$

$\therefore \Delta\text{ABO} /// \Delta\text{OBC}$ (AAA)

$\therefore \frac{\text{AB}}{\text{OB}} = \frac{\text{BO}}{\text{BC}}$

$\therefore \text{AB} = \frac{\text{OB}^2}{\text{BC}}$

QUESTION 10

(a) Let $\text{AB} = 4k$ and $\text{BC} = 7k$

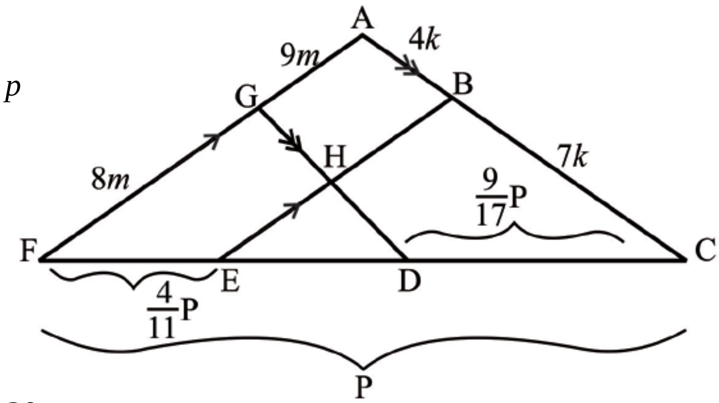
$\therefore \frac{\text{FE}}{\text{FC}} = \frac{\text{AB}}{\text{AC}} = \frac{4}{11}$; (Proportionality theorem OR using theorem on diagram) (3)

(b) Let $\text{AG} = 9m$ and $\text{AF} = 17m$

$\frac{\text{CD}}{\text{DF}} = \frac{\text{AG}}{\text{GF}} = \frac{9}{8}$ (2)

(c) If $\text{FC} = p$ then $\text{ED} = p - \frac{9}{17}p - \frac{4}{11}p$

$\text{ED} = \frac{20}{187}p$



The length of ED in kilometres is $\frac{20}{187} \times 374$ km $= 40$ kilometres.

It will take 2 000 hours to build the track from E to D.

OR

Alternate:

Let $\text{FE} = 4p$ and $\text{EC} = 7p$

$\text{FD} = 8m$ and $\text{DC} = 9m$

$\therefore 11p = 374 \quad \therefore p = 34$

$17m = 374 \quad \therefore m = 22$

$\therefore \text{DC} = 374 - 4p - 9m$

$= 40$ km

$\therefore$ 2 000 hours

OR

Alternate:

$\text{FE} = \frac{4}{11}(374) = 136$

$\text{CD} = \frac{9}{17}(374) = 198$

$\therefore \text{ED} = 374 - 136 - 198$

$= 40$ km

$\therefore$ 4 hours $\rightarrow 40 \times 50$

$= 2\ 000$ hours

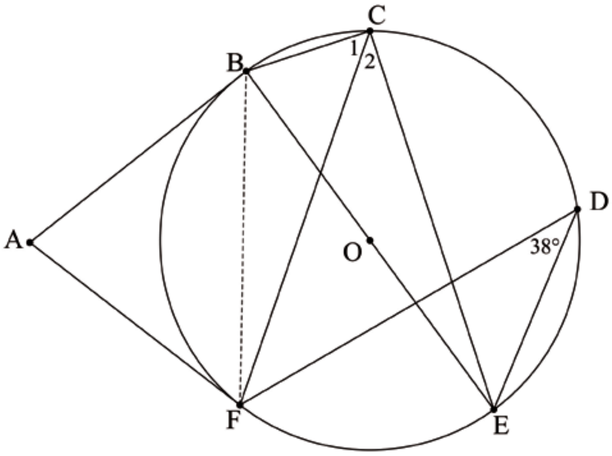
QUESTION 11

(a) $\hat{C}_2 = \hat{D}$ (angles in same segment)

$\hat{C}_1 + \hat{C}_2 = 90°$ (angle in semi-circle)

$\therefore \hat{C}_1 + \hat{D} = 90°$

(b) Construction: Chord BF



$\hat{C}_1 = 90° - \hat{C}_2 = 52°$

$A\hat{F}B = \hat{C}_1$ (tan chord theorem)

$A\hat{B}F = 52°$ (tan chord theorem)

$\therefore B\hat{A}F = 76°$ (angles of a Δ)

QUESTION 12

(a) Area of $\Delta\text{ADC} = \frac{1}{2} \times 6 \times 6 \times \sin 130°$

$= 13{,}8$

(b) $\text{A}\hat{\text{B}}\text{C} = 50°$ (opp Δ's cyclic quad)

$\text{A}\hat{\text{B}}\text{D} = \text{D}\hat{\text{B}}\text{C}$ (Equal chords; subtend equal angles)

$\therefore \text{D}\hat{\text{B}}\text{C} = 25°$

(c) $\text{BC} = 12$ (line from centre $\perp$ chord)

$\frac{\sin \text{B}\hat{\text{D}}\text{C}}{12} = \frac{\sin 25°}{6}$ (sin rule)

$\therefore \sin \text{B}\hat{\text{D}}\text{C} = 0{,}845...$

$\text{B}\hat{\text{D}}\text{C} = 57{,}7°$

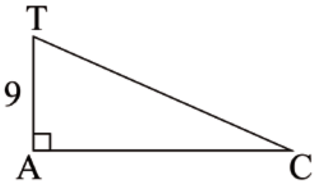
$\therefore \text{B}\hat{\text{D}}\text{C} = 180 - 57{,}7°$

$= 122{,}3°$

$\therefore \theta = 180° - 25° - 122{,}3$ (angles of Δ)

$\theta = 32{,}7°$

(d)



$\text{AC}^2 = 6^2 + 6^2 - 2(6)(6)\cos 130°$

$\text{AC}^2 = 118{,}28...$

$\therefore \text{AC} = 10{,}875...$

$\therefore \tan \text{T}\hat{\text{C}}\text{A} = \frac{9}{10{,}875...}$

$\therefore \text{T}\hat{\text{C}}\text{A} = 39{,}6°$

Total: 150 marks