ANALYTICAL GEOMETRY TEST

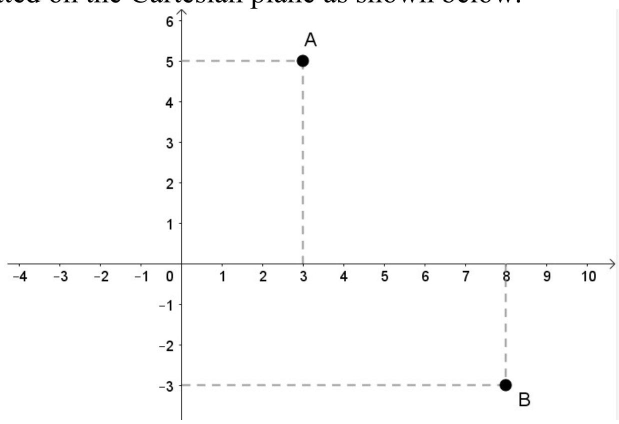
Grade 10
Mathematics
Marks: **50**
Time: **1 hour**

Name: **MEMORANDUM**

REFRACTION
www.refractions.co.za

QUESTION 1

Points A and B are plotted on the Cartesian plane as shown below.



1.1 Determine the distance AB. (3) S1501

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \checkmark$$

$$= \sqrt{(3-8)^2 + (5-(-3))^2} \checkmark$$

$$= \sqrt{89} \checkmark$$

1.2 What is the gradient of the line passing through points A and B? (3) S1504

$$m = \frac{y_A - y_B}{x_A - x_B} \checkmark$$

$$= \frac{5-(-3)}{3-8} \checkmark$$

$$= -\frac{8}{5} \checkmark$$

1.3 Determine the coordinates of the midpoint M of the line segment joining A and B. (3) S1503

$$M\left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right) \checkmark$$

$$= M\left(\frac{3+8}{2} ; \frac{5-3}{2}\right) \checkmark$$

$$= M\left(\frac{11}{2} ; 1\right) \checkmark$$

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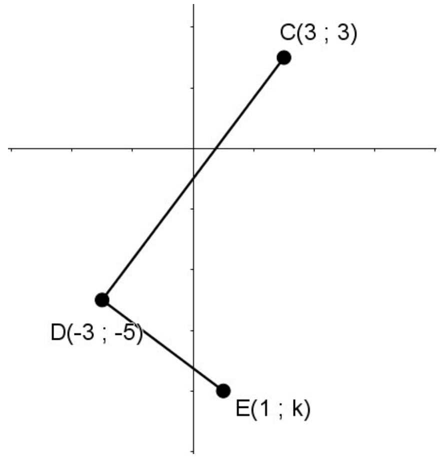
QUESTION 2

2.1 In the diagram below, $C(3\,;3)$, $D(-3\,;\,-5)$ and $E(1\,;k)$ are three points in the Cartesian plane.



2.1.1 Calculate the length of CD. (3) | S1501

$$CD = \sqrt{(x_C - x_D)^2 + (y_C - y_D)^2} ✓$$

$$= \sqrt{(3-(-3))^2 + (3-(-5))^2} ✓$$

$$= 10 ✓$$

2.1.2 Calculate the gradient of CD. (3) | S1504

$$m = \frac{y_C - y_D}{x_C - x_D} ✓$$

$$= \frac{3-(-5)}{3-(-3)} ✓$$

$$= \frac{4}{3} ✓$$

2.1.3 Determine the value of k if $C\hat{D}E = 90°$. (4) | S1504

$$m_{DE} = -\frac{3}{4} ✓$$

$$m = \frac{y_D - y_E}{x_D - x_E}$$

$$-\frac{3}{4} = \frac{k-(-5)}{1-(-1)} ✓$$

$$\frac{-3 \times 4}{4} = k + 5 ✓$$

$$-3 - 5 = k$$

$$k = -8 ✓$$

2.1.4 If $k = -8$, determine the coordinates of M, the midpoint of CE. (3) S1503

$M\left(\frac{x_C + x_E}{2} ; \frac{y_C + y_E}{2}\right)$ ✓

$= M\left(\frac{3+1}{2} ; \frac{3+(-8)}{2}\right)$ ✓

$= M\left(2 ; -\frac{5}{2}\right)$ ✓

2.1.5 Determine the coordinates of point F such that the quadrilateral CDEF is a rectangle. (4) S1505

$x_F - x_E = x_C - x_D$ ✓

$x_F - 1 = 3 - (-3)$

$x_F = 6 + 1$

$= 7$ ✓

$y_F - y_E = y_C - y_D$

$y_F - (-8) = 3 - (-5)$

$y_F + 8 = 8$

$y_F = 0$ ✓

$F(7 ; 0)$ ✓

OR

$M\left(\frac{3+1}{2} ; \frac{3+(-8)}{2}\right) = M\left(2 ; -\frac{5}{2}\right)$ ✓

$M\left(2 ; -\frac{5}{2}\right) = M\left(\frac{-3+x}{2} ; \frac{-5+y}{2}\right)$

$\therefore \frac{-3+x}{2} = 2$

$x = 7$ ✓

$\therefore \frac{-5+y}{2} = -\frac{5}{2}$

$y = 0$ ✓

$F(7 ; 0)$ ✓

2.2 G is the point $(0 ; -4)$. The point H lies in the second quadrant and has coordinates $(x ; 2)$. If the length of GH is $\sqrt{61}$ units, calculate the value of x. (4) S1502

$GH = \sqrt{(x_G - x_H)^2 + (y_G - y_H)^2}$

$\sqrt{61} = \sqrt{(x-0)^2 + (2-(-4))^2}$ ✓

$61 = x^2 + 36$ ✓

$x^2 = 61 - 36$

$x^2 = 25$

$x = \pm 5$ ✓

Second quadrant $\therefore x = -5$ ✓

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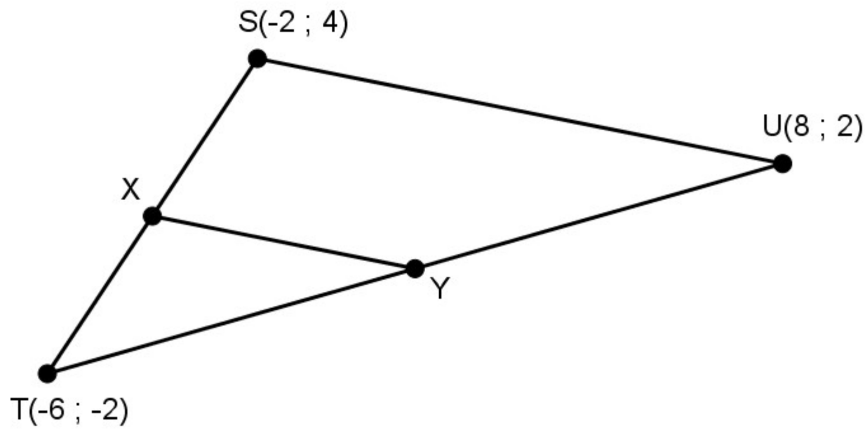
QUESTION 3

In the diagram below, the coordinates of ΔSTU are given as $S(-2\,;4)$, $T(-6\,;\,-2)$ and $U(8\,;2)$.
X and Y are the midpoints of ST and TU respectively.



3.1 Calculate the coordinates of X and Y. (6) S1503

$$X\left(\frac{x_S + x_T}{2}\,;\,\frac{y_S + y_T}{2}\right)$$ ✓

$$= X\left(\frac{-2 + (-6)}{2}\,;\,\frac{4 + (-2)}{2}\right)$$ ✓

$$= X(-4\,;\,1)$$ ✓

$$Y\left(\frac{x_U + x_T}{2}\,;\,\frac{y_U + y_T}{2}\right)$$ ✓

$$= Y\left(\frac{8 + (-6)}{2}\,;\,\frac{2 + (-2)}{2}\right)$$ ✓

$$= Y(1\,;0)$$ ✓

3.2 Show that: S1504

3.2.1 $XY \parallel SU$ (4)

$$m_{SU} = \frac{4-2}{-2-8}$$

$$= -\frac{1}{5}$$ ✓

$$m_{XY} = \frac{1-0}{-4-1}$$

$$= -\frac{1}{5}$$ ✓

$$m_{SU} = m_{XY}$$ ✓

$$\therefore SU \parallel XY$$ ✓

3.2.2 $XY = \frac{1}{2}SU$ (4)

$XY = \sqrt{(-4-1)^2 + (1-0)^2}$ ✓

$= \sqrt{26}$ ✓

$SU = \sqrt{(-2-8)^2 + (4-2)^2}$

$= 2\sqrt{26}$ ✓

$\therefore XY = \frac{1}{2}SU$ ✓

3.3 Calculate, to two decimal places, the perimeter of ΔSTU. (6) S1502

$SU = 2\sqrt{26}$ ✓

$ST = \sqrt{(-2-(-6))^2 + (4-(-2))^2}$ ✓

$= 2\sqrt{13}$ ✓

$UT = \sqrt{(8-(-6))^2 + (2-(-2))^2}$ ✓

$= 2\sqrt{53}$ ✓

$Perimeter = 2\sqrt{26} + 2\sqrt{13} + 2\sqrt{53}$

$= 31{,}97$ ✓

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Total: 50 Marks