assessment matters

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2016

MATHEMATICS: PAPER I

Time: 3 hours | 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages and an Information Sheet of 2 pages (i–ii). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved, non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. Diagrams are not necessarily drawn to scale.
9. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) Solve for x:

(1) $\frac{4x}{2} - \frac{2x+1}{3} = 5$ (2)

(2) $(x-5)(x-6) \leq 56$ (5)

(b) Given: $f(x) = 2(x+2)^2 - 8$

Sketch the graph of f.
Show the turning point and intercepts with the axes. (5)

(c) Given: $g(x) = \frac{4}{x+1} + 2$

(1) Write down the equations of the vertical and horizontal asymptotes. (2)

(2) Determine the point(s) of intersections of the graphs of $g(x)$ and $y = x$. (4)

(d) Given the equation $x^2 + c = 0$, where $-2 < c < 5$.

Give two values of c for which the roots of the equation are unequal and rational. (2)

(e) The roots of a quadratic equation are given by $x = \frac{-1 \pm \sqrt{3-k}}{2}$.

Determine the value(s) of k for which the roots will be non-real. (2)

[22]

QUESTION 2

(a) Given: $3x = -\sqrt{6x-1}$

(1) A student claims that a solution to the given equation is $x = \frac{1}{3}$.

Show that the student's claim is incorrect. (2)

(2) Show that the equation has no real solution. (4)

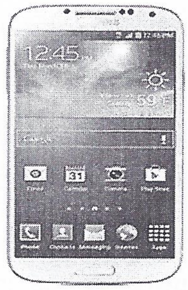
(b) Given: $7^{x+a} + 3\left(7^{x+a}\right) = 28\left(7^{a^2}\right)$

Solve for x in terms of a, leaving the answer in its simplest form. (3)

[9]

QUESTION 3

(a) A cellular phone has a marked price of R4 800. During a sale, a discount of 13,5% was offered. What is the selling price of the phone?



[Source: <www.juzdeals.com>]

(2)

(b) A small business owner was granted a loan for purchasing one hundred cellular phones. He paid R4 800 less a discount of 13,5% for each phone.

The loan had to be repaid in instalments at the end of each month at an interest rate of 7% per annum compounded monthly.

Calculate the monthly instalments if the loan repayment period was 5 years. (4)

[6]

QUESTION 4

[Source: <www.coolpctips.com>]

A school issued new laptops to each of its 110 employees at the beginning of the year. The school was advised to set up a sinking fund to ensure that there would be enough money to replace them at the end of the 5^{th} year.

The following applies:

- They paid R6 000 for each laptop.
- The laptops depreciate at 15% per annum on a reducing balance basis.
- The supplier will buy back all 110 laptops at the end of 5 years at the depreciated value.
- Inflation is estimated to be worked out at 6% per annum over the five-year period.
- The sinking fund is set up so that all payments will receive 12% interest per annum compounded monthly.

(a) Determine the amount of money required at the end of 5 years to replace the laptops. (4)

(b) Determine the monthly payments that should be made into the sinking fund to ensure that all 110 laptops can be replaced at the end of 5 years. (4)

[8]

QUESTION 5

(a) Given: $\sum_{x=1}^{y} 5x + 2 = 36$. Determine y. (3)

(b) An arithmetic sequence is given as: $(2p+14)\,;3p\,;(p+7)\,;...$

(1) Determine p. (2)

(2) Hence determine the sum of the first 38 terms. (3)

(c) The following sequence is given:

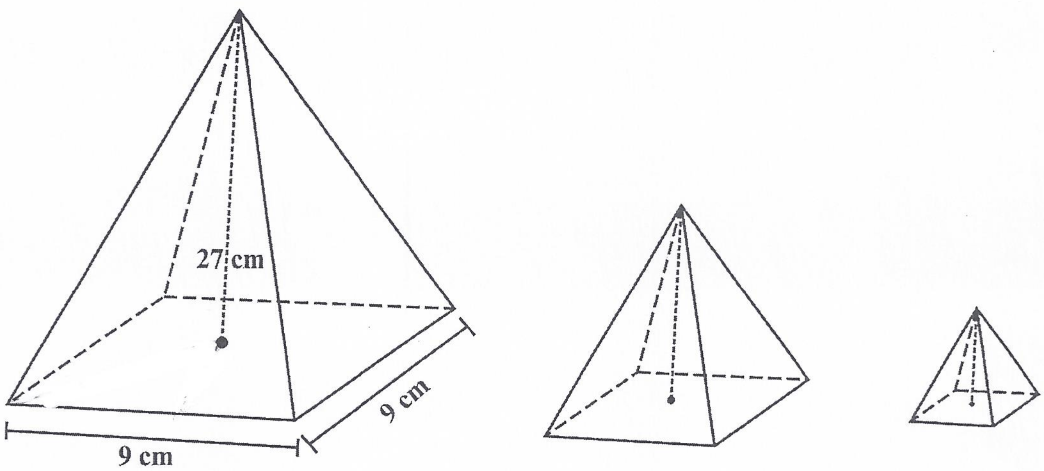
$$\frac{2^3-1}{1};\frac{3^3-1}{2};\frac{4^3-1}{3};\frac{5^3-1}{4}\ ...$$

Given that the sequence is quadratic, determine the n^{th} term. Simplify your answer as far as possible. (4)

(d) The sum of the first n terms of a geometric sequence 9 + 6 + 4 + … is greater than 25. Calculate the smallest value of n. (6)

(e) A solid right pyramid with a square base has a perpendicular height of 27 cm. The base has a length of 9 cm. This pyramid is replicated under the following constraints:

The base area and perpendicular height of each replica is one third of the previous one.



Determine the total volume of all the pyramids replicated, if this replication continues indefinitely.

Useful formula: Volume of a pyramid $= \frac{1}{3}$ A × H. (5)

[23]

QUESTION 6

(a) Given $f(x) = 3x^2 + 2x$, determine $f'(x)$ from first principles. (5)

(b) Differentiate with respect to x: $y = -\frac{1}{x} + \sqrt{x}$. (4)

[9]

77 marks

SECTION B

QUESTION 7

Sketch a possible graph of $y = f(x) = ax^3 + bx^2 + cx + d$ if:

$f''(x) < 0$ for $x > 0$ and the graph has a point of inflection at (0;1).

[3]

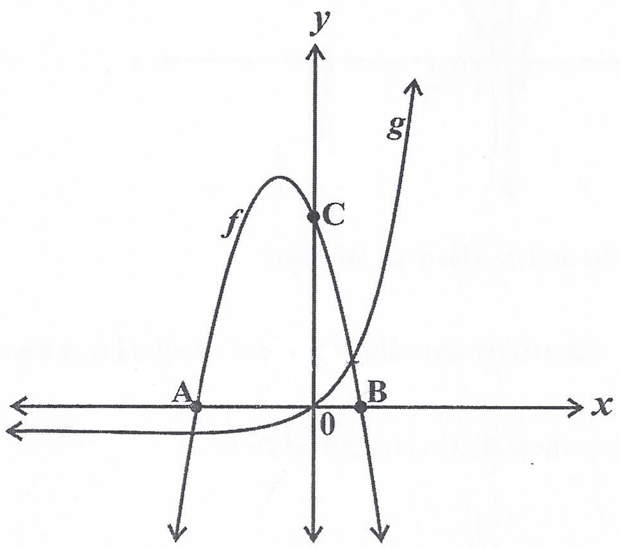
QUESTION 8

The sketch represents the graphs of the functions $f(x) = ax^2 + bx + c$ and $g(x) = d^x + q$.

The x-intercepts of f are A (–3;0) and B (1;0).

The y-intercept of f is C (0;6).

The graph of g passes through the origin and the point (1;2).



(a) Use the graph to determine the values of x where $f'(x) \cdot g(x) < 0$ (4)

(b) Determine d and q. (4)

(c) Determine g^{-1}, the inverse of g, in the form $y = \ldots$ (3)

(d) State the domain of g^{-1}. (2)

(e) Determine a, b and c. (4)

(f) Draw a sketch graph of f^{-1}, the inverse of f. Show the turning point and the intercepts with the axes. (5)

(g) Determine the values of k for which $f(x) + k = g(x)$ has two roots that are opposite in sign. (2)

[24]

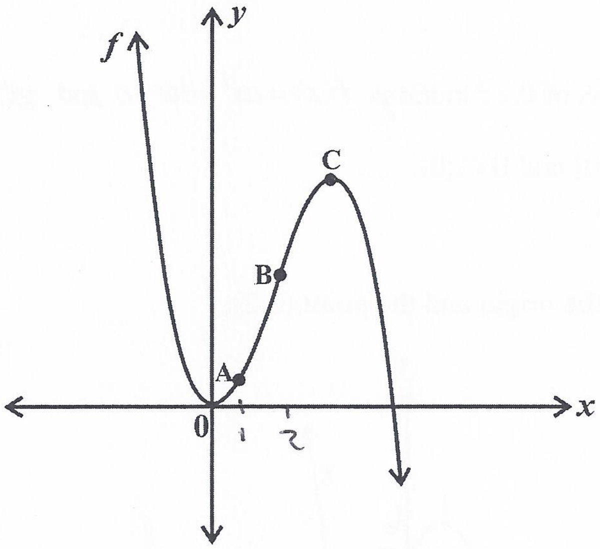
QUESTION 9

The graph of $f(x) = ax^3 + bx^2$ is sketched below.

The x-coordinates at A and B are 1 and 2 respectively.

The average gradient of f between A and B is 5,5.

The equation of the tangent to the curve of f at $x = 6$, is $y = -18x + c$.



(a) Determine the values of a and b. Show all working. (8)

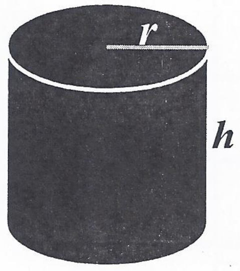
(b) If $f(x) = \frac{-1}{2}x^3 + 3x^2$, determine the values of x for which $f(x)$ is increasing. (4)

(c) Determine the interval(s) where $f(x)$ is concave down. (3)

[15]

QUESTION 10

A hat box, in the shape of a circular cylinder, is to be constructed in such a manner that the sum of its height and the radius is 9 units. Determine the radius for which the cylinder has the largest possible volume.



Useful formula: Volume of a cylinder = $\pi r^2 h$

[7]

QUESTION 11

(a) The lengths of 80 worms, in centimetres, are recorded in the table below:

[Source: <www.wigglywigglers.co.uk>]

Length in cm	(0;5]	(5;10]	(10;15]	(15;20]	(20;25]
Frequency	9	21	25	17	8

Two worms were chosen at random.

Find the probability that:

(1) both worms were longer than 5 cm but less than or equal to 15 cm. (3)

(2) one worm was 5 cm or less and the other was longer than 15 cm. (4)

(b) The word *CATALYST* is given:

Determine the number of different ways that the letters can be arranged. (3)

(c) There are **6 red** cards and **1 black** card in a box. Busi and Khanya take turns to draw a card at random from the box, with Busi being the first one to draw. The first person who draws the black card will win the game. (Assume that the game can go on indefinitely.)

If the cards are drawn **with** replacement, determine the probability that Khanya will win, showing all working. (7)

[17]

QUESTION 12

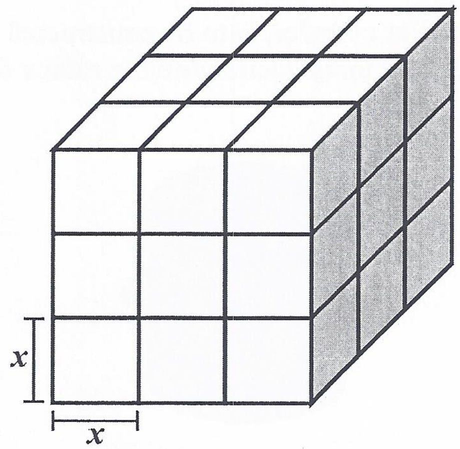


Figure 1

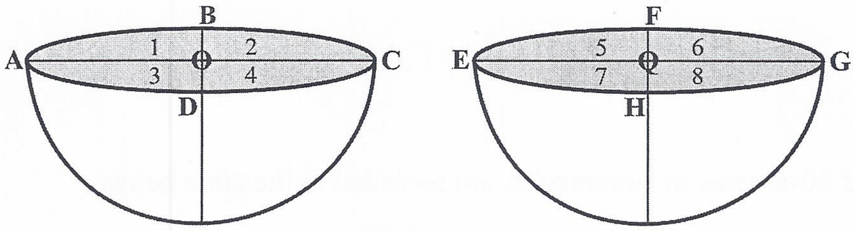


Figure 2

A cube (see Figure 1) is made up of 27 identical smaller cubes, each having side length x. A sphere of radius x (see Figure 2) is cut into eight identical "quarter hemispheres". The eight corners of the cube are removed and replaced with these eight pieces, to form a paperweight as shown in Figure 3.

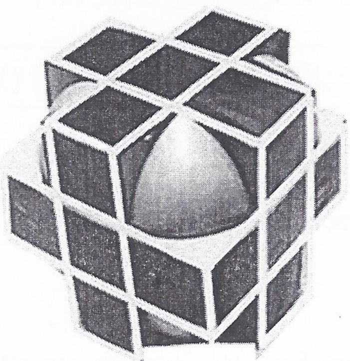


Figure 3

If the total surface area of the paperweight is 28 mm^2, determine the size of x.

Note: figures are not drawn to scale.

Useful formulae: Surface Area of Sphere = $4\pi r^2$

[7]

73 marks

Total: 150 marks