



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL/NASIONALE
SENIOR
CERTIFICATE/SERTIFIKAAT**

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2016

MEMORANDUM

MARKS/PUNTE: 150

**This memorandum consists of 26 pages.
*Hierdie memorandum bestaan uit 26 bladsye.***

NOTE:

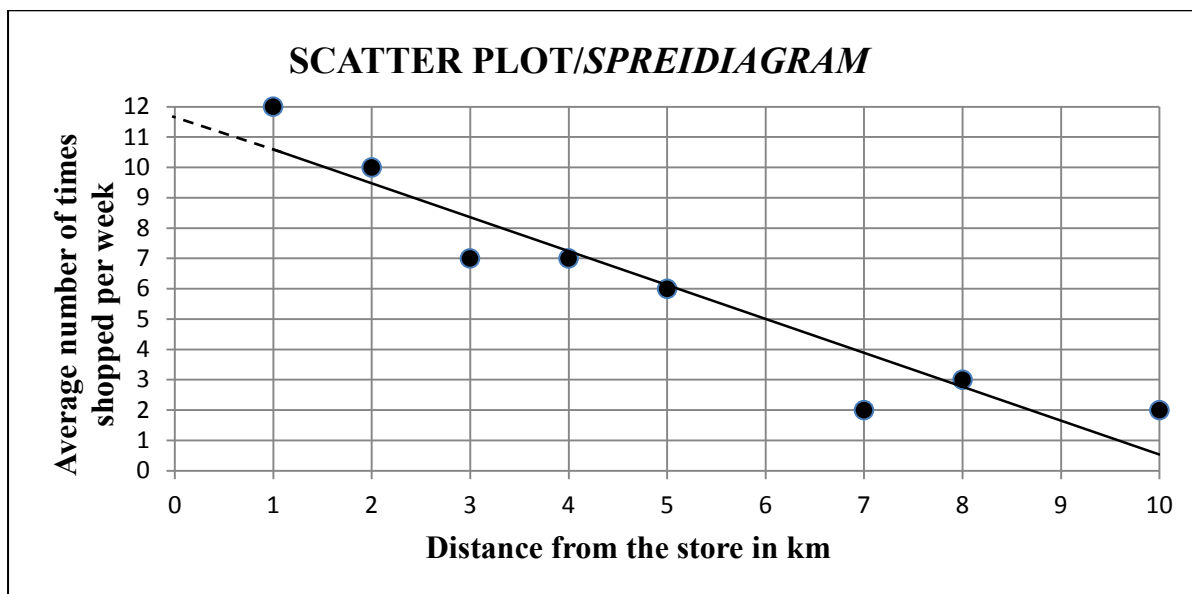
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/VRAAG 1

| | | | | | | | | |
|--|----|----|---|---|---|---|---|----|
| Distance from the store in km <i>Afstand vanaf die winkel in km</i> | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 10 |
| Average number of times shopped per week <i>Gemiddelde aantal keer wat kopers die winkel per week besoek</i> | 12 | 10 | 7 | 7 | 6 | 2 | 3 | 2 |



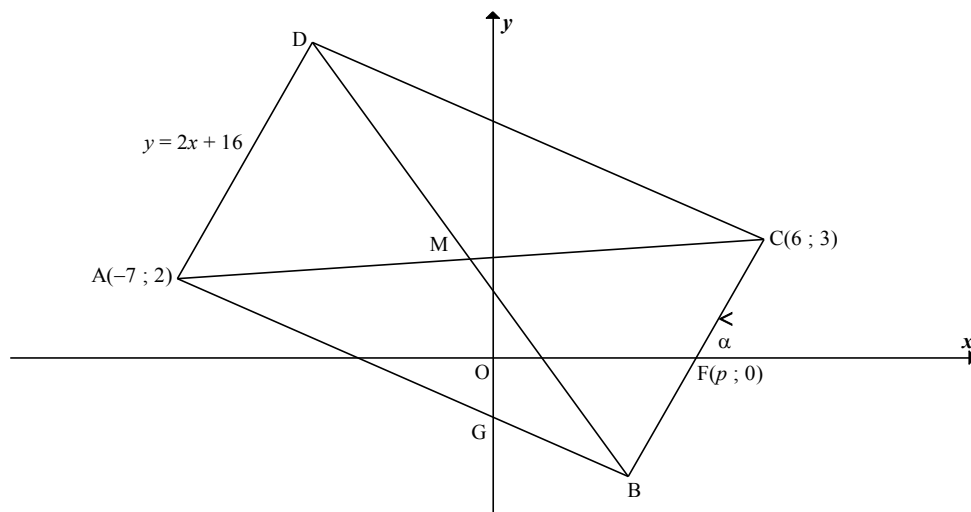
| | | | |
|-----|--|--|-------------------|
| 1.1 | Strong/ <i>Sterk</i> | ✓ | (1) |
| 1.2 | -0,95 (-0,9462...) | ✓ | (1) |
| 1.3 | $a = 11,71$ (11,7132...) $b = -1,12$ (-1,1176...) $\hat{y} = -1,12x + 11,71$ | ✓ value of a ✓ value of b ✓ equation/vgl | (3) |
| 1.4 | $\hat{y} = -1,12(6) + 11,71$ = 5 times | ✓ substitution ✓ answer | (2) |
| 1.5 | On scatter plot/ <i>Op spreidiagram</i> | ✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) | (2) [9] |

QUESTION/VRAAG 2

| 2.1 | Positively skewed OR skewed to the right/ <i>positief skeef OF skeef na regs</i> | ✓ answer (1) | | | | | | | | | | | | |
|----------------------------|--|---|---|--------------------|----|--------------------|----|--------------------|-----|--------------------|-----|--------------------|-----|--|
| 2.2 | Range/ <i>Omvang</i> = $2,21 - 1,39 = 0,82$ m | ✓ subtract values ✓ answer (2) | | | | | | | | | | | | |
| 2.3 | <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Intervals <i>Klasse</i></th> <th style="text-align: center;">Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">$1,3 \leq x < 1,5$</td> <td style="text-align: center;">24</td> </tr> <tr> <td style="text-align: center;">$1,5 \leq x < 1,7$</td> <td style="text-align: center;">95</td> </tr> <tr> <td style="text-align: center;">$1,7 \leq x < 1,9$</td> <td style="text-align: center;">133</td> </tr> <tr> <td style="text-align: center;">$1,9 \leq x < 2,1$</td> <td style="text-align: center;">156</td> </tr> <tr> <td style="text-align: center;">$2,1 \leq x < 2,3$</td> <td style="text-align: center;">160</td> </tr> </tbody> </table> | Intervals <i>Klasse</i> | Cumulative frequency <i>Kumulatiewe frekwensie</i> | $1,3 \leq x < 1,5$ | 24 | $1,5 \leq x < 1,7$ | 95 | $1,7 \leq x < 1,9$ | 133 | $1,9 \leq x < 2,1$ | 156 | $2,1 \leq x < 2,3$ | 160 | ✓95 , 133, 156 ✓160 (2) |
| Intervals <i>Klasse</i> | Cumulative frequency <i>Kumulatiewe frekwensie</i> | | | | | | | | | | | | | |
| $1,3 \leq x < 1,5$ | 24 | | | | | | | | | | | | | |
| $1,5 \leq x < 1,7$ | 95 | | | | | | | | | | | | | |
| $1,7 \leq x < 1,9$ | 133 | | | | | | | | | | | | | |
| $1,9 \leq x < 2,1$ | 156 | | | | | | | | | | | | | |
| $2,1 \leq x < 2,3$ | 160 | | | | | | | | | | | | | |
| 2.4 | <p style="text-align: center;">OGIVE/OGIEF</p> | ✓ upper limits / <i>boonste limiete</i> ✓ cum <i>f</i> / <i>kum f</i> ✓ shape / <i>vorm</i> ✓ grounded <i>geanker</i> (4) | | | | | | | | | | | | |
| 2.5 | method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69) | ✓ method ✓ answer (2) | | | | | | | | | | | | |
| 2.6.1 | The mean would change by 0,1 m <i>Die gemiddelde sal met 0,1 m verander</i> | ✓ answer (1) | | | | | | | | | | | | |
| 2.6.2 | No influence/change as there is no difference in variation of data./ <i>Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.</i> | ✓ answer (1) | | | | | | | | | | | | |

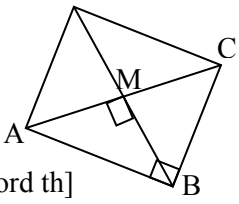
[13]

QUESTION/VRAAG 3

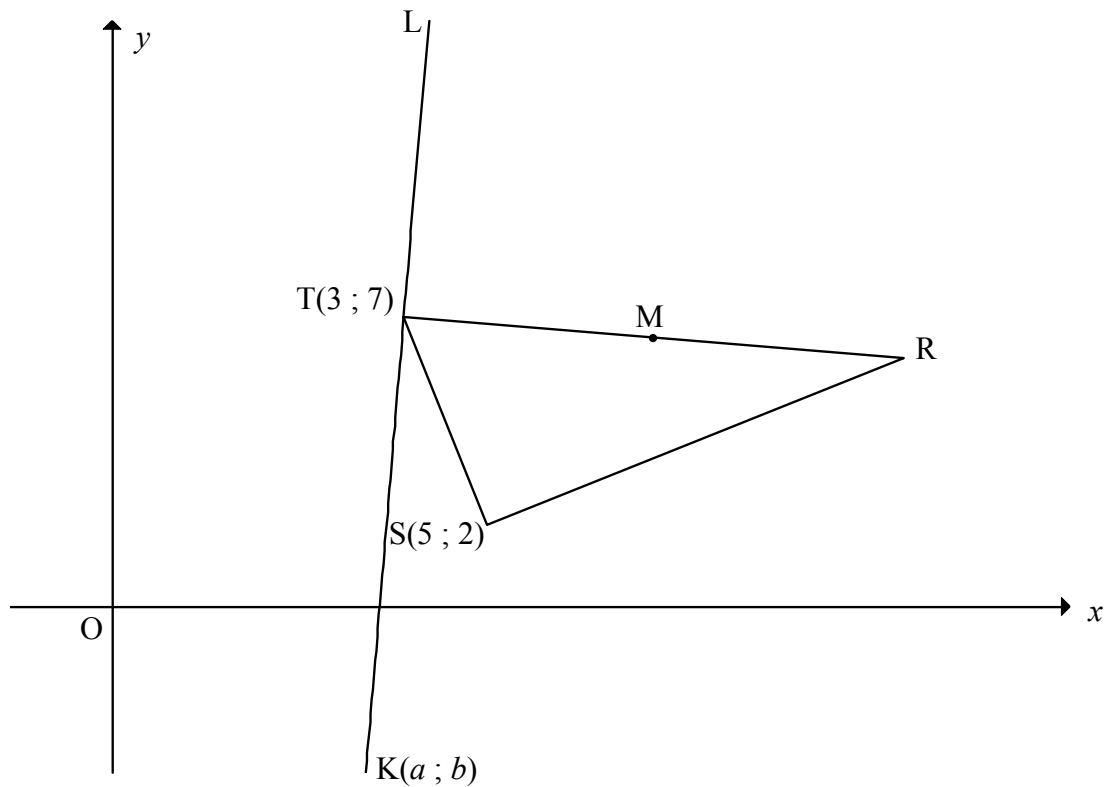


| | | |
|------------|---|--|
| <p>3.1</p> | <p>M = Midpt of AC [diags of rectangle bisect/ hoekl v reghoek halveer]</p> $= M\left(\frac{-7+6}{2}; \frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2}; \frac{5}{2}\right)$ | <p>✓ x-value of M ✓ y-value of M (2)</p> |
| <p>3.2</p> | $m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ <p>OR/OF</p> $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$ | <p>✓ answer (1)</p> <p>✓ answer (1)</p> |
| <p>3.3</p> | $m_{AD} = m_{BC} \text{ [AD BC]}$ $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ <p>OR/OF</p> $y - y_1 = 2(x - x_1)$ <p>C(6;3)</p> $y - 3 = 2(x - 6)$ $\therefore y = 2x - 9$ <p>but y = 0</p> $\therefore x = 4\frac{1}{2} = p$ <p>OR/OF</p> | <p>✓ $m_{BC} = 2$</p> <p>✓ equating</p> <p>✓ answer (3)</p> <p>✓ $m_{BC} = 2$</p> <p>✓ substituting (6; 3)</p> <p>✓ answer (3)</p> |

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| | $y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \quad \therefore p = \frac{9}{2}$ | ✓ $m_{BC} = 2$ ✓ substituting ✓ answer (3) |
| 3.4 | DB = AC [diag of rectangle = / <i>hoekl v reghoek</i> =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6 + 7)^2 + (3 - 2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore DB = \sqrt{170}$ or 13,04 | ✓ substitution ✓ length of AC ✓ AC = BD (3) |
| 3.5 | $\tan \alpha = m_{BC} = 2$ $\therefore \alpha = 63,43^\circ$ | ✓ $\tan \alpha = m_{BC}$ ✓ $\alpha = 63,43^\circ$ (2) |
| 3.6 | In quadrilateral OFBG: $\widehat{OFB} = 63,43^\circ$ [vert opp \angle s/ <i>regoorst</i> \angle e] $\widehat{FOG} = \widehat{GBF} = 90^\circ$ $\therefore \widehat{OGB} = 360^\circ - [90^\circ + 90^\circ + 63,43^\circ]$ [sum \angle s quad/ <i>som</i> \angle e vierh = 360°] $\therefore \widehat{OGB} = 116,57^\circ$ OR/OF $m_{AB} = -\frac{1}{2}$ $90^\circ + \widehat{OGA} = 153,43^\circ$ $\therefore \widehat{OGA} = 63,43^\circ$ $\widehat{OGB} = 180^\circ - 63,43^\circ$ $= 116,57^\circ$ OR/OF $\widehat{FOG} = \widehat{GBF} = 90^\circ$ \therefore GOFB is cyc quad $\widehat{OGB} = 180^\circ - 63,43^\circ$ [\angle s of cyc quad = 180°] $= 116,57^\circ$ OR/OF $\widehat{OFB} = 63,43^\circ$ $\widehat{XOG} = \widehat{FBG} = 90^\circ$ \therefore OGBF is a cyclic quad $\therefore \widehat{OGB} = 180^\circ - 63,43^\circ$ $\widehat{OGB} = 116,57^\circ$ | ✓ size of \widehat{OFB} ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) |

| | | |
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| <p>3.7</p> | <p>$M\left(-\frac{1}{2}; \frac{5}{2}\right)$ is the centre/<i>is die middelpunt</i></p> <p>$r = \frac{\sqrt{170}}{2} = \text{radius}$ [BD is diameter/<i>middellyn</i>]</p> <p>$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{\sqrt{170}}{2}\right)^2 = \frac{85}{2} = 42,5$</p> | <p>✓ M is centre</p> <p>✓ $r = \frac{\sqrt{170}}{2}$</p> <p>✓ equation</p> <p>(3)</p> |
| <p>3.8</p> | <p>$\hat{C}BM = \hat{B}AM = 45^\circ$ [diag of square bisect \angles/<i>hoekl v vierk halv \anglee</i>] $\therefore BC$ will be a tangent [converse tan chord th/<i>omgekeerde raakl-koordst</i>] OR/OF</p> <p>$\hat{A}MB = 90^\circ$ [diag of square bisect \perp] $\therefore AB$ is diameter $BC \perp AB$ $\therefore BC$ is tangent [line \perp radius <i>or</i> converse tan-chord th]</p>  | <p>✓S</p> <p>✓R</p> <p>(2)</p> <p>✓S</p> <p>✓R</p> <p>(2)</p> <p>[19]</p> |

QUESTION/VRAAG 4



| | | |
|-----|---|---|
| 4.1 | \angle in semi circle/ \angle at centre = $2\angle$ on circle \angle in halfsirkel / \angle by middelpt = $2\angle$ op sirkel | \checkmark R <div style="text-align: right;">(1)</div> |
| 4.2 | $m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$ | \checkmark substitution \checkmark m_{TS} <div style="text-align: right;">(2)</div> |
| 4.3 | $m_{TS} \times m_{RS} = -1$ [TS \perp SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ OR/OF | \checkmark m_{RS} \checkmark substitution m and (5 ; 2) \checkmark equation <div style="text-align: right;">(3)</div> |

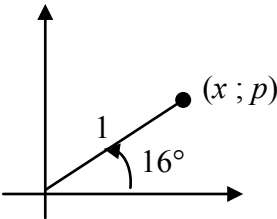
| | | |
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| | $m_{TS} \times m_{RS} = -1 \quad [TS \perp SR]$ $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$ | <p>✓ m_{RS}</p> <p>✓ substitution m and $(5 ; 2)$</p> <p>✓ equation (3)</p> |
| <p>4.4.1</p> | $r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ <p>OR/OF</p> $TM = \sqrt{(3-9)^2 + \left(7-6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ | <p>✓ r</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> |
| <p>4.4.2</p> | $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9 \quad \text{and} \quad \frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: full marks Answer only: only 1 coordinate correct (1 mark)</p> </div> <p>OR/OF</p> $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9+6 ; 6\frac{1}{2}-\frac{1}{2}\right) = R(15 ; 6)$ <p>OR/OF</p> | <p>✓ M</p> <p>✓ x coordinate ✓ y coordinate (3)</p> <p>✓ M</p> <p>✓ x coordinate ✓ y coordinate (3)</p> |

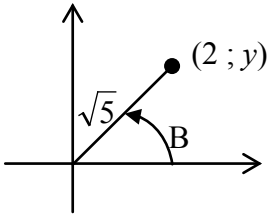
| | | |
|--------------|--|---|
| | $m_{TM} = \frac{9-3}{6\frac{1}{2}-7} = -\frac{1}{12}$ $TM : 7 = -\frac{1}{12}(3) + c \quad y = -\frac{1}{12}x + \frac{29}{4} \quad \dots\dots(1)$ $SR : y = \frac{2}{5}x \quad \dots\dots(2)$ $\frac{2}{5}x = -\frac{1}{12}x + \frac{29}{4}$ $\frac{29}{60}x = \frac{29}{4}$ $\therefore x = 15$ $\therefore y = \frac{2}{5}(15) = 6$ | <p>✓ equating</p> <p>✓ x coordinate</p> <p>✓ y coordinate</p> <p>(3)</p> |
| <p>4.4.3</p> | $ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $ST = \sqrt{(5-3)^2 + (2-7)^2}$ $ST = \sqrt{4+25} = \sqrt{29}$ $\sin R = \frac{TS}{TR} = \frac{\sqrt{29}}{\sqrt{145}} \text{ or } \frac{\sqrt{5}}{5} \text{ or } \frac{1}{\sqrt{5}} \text{ or } 0,45$ <p>OR/OF</p> $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ $\text{area of } \Delta TSR = \frac{1}{2}(\sqrt{29})(2\sqrt{29}) = 29$ $29 = \frac{1}{2}(\sqrt{145})(2\sqrt{29}) \sin R$ $\sin R = \frac{\sqrt{5}}{5} \text{ or } \frac{1}{\sqrt{5}}$ | <p>✓ substitution</p> <p>✓ answer</p> <p>✓ ratio</p> <p>(3)</p> <p>✓ area</p> <p>✓ rule</p> <p>✓ ratio</p> <p>(3)</p> |
| <p>4.4.4</p> | $m_{TR} = \frac{7-6}{3-9} = -\frac{1}{12} \quad \text{OR/OF} \quad m_{TR} = \frac{7-6}{3-15} = -\frac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $m_{KTL} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x - 3)$ $y = 12x - 29$ <p>substitute K(a; b):</p> $b = 12a - 29$ <p>OR/OF</p> | <p>✓ $m_{TR} = -\frac{1}{12}$</p> <p>✓ $m_{KTL} = 12$</p> <p>✓ $y = 12x - 29$</p> <p>(3)</p> |

| | | |
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| | $m_{TR} = \frac{7 - 6\frac{1}{2}}{3 - 9} = -\frac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $\frac{b - 7}{a - 3} = 12$ $b - 7 = 12(a - 3)$ $b = 12a - 29$ <p>OR/OF</p> $KR^2 = TR^2 + TK^2$ $(a - 15)^2 + (b - 6)^2 = (15 - 3)^2 + (6 - 7)^2 + (a - 3)^2 + (b - 7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$ | $\checkmark m_{TR} = -\frac{1}{12}$ $\checkmark m_{KTL} = 12$ $\checkmark \text{substitution}$ $(3 ; 7) \text{ \& } (a ; b)$ <p style="text-align: right;">(3)</p> $\checkmark \text{subst into Pyth}$ $\checkmark \text{multiplication}$ $\checkmark \text{simplification}$ <p style="text-align: right;">(3)</p> |
| <p>4.4.5</p> | $TK = TR$ $\sqrt{(a - 3)^2 + (b - 7)^2} = \sqrt{145}$ $(a - 3)^2 + (b - 7)^2 = 145$ <p>Substitute $b = 12a - 29$ [from 4.4.4]</p> $(a - 3)^2 + (12a - 29 - 7)^2 = 145$ $(a - 3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \frac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2 \text{ or } a = 4$ $\therefore b = 12(2) - 29 = -5 \quad \text{or} \quad b = 12(4) - 29 = 19$ $\therefore K(2 ; -5)$ <p>OR/OF</p> | $\checkmark \text{substitution into distance formula}$ $\checkmark \text{substitution of } b = 12a - 29$ $\checkmark \text{standard form}$ $\checkmark \text{subst into formula or factorise}$ $\checkmark \text{values of } a$ $\checkmark \text{value of } b$ <p style="text-align: right;">(6)</p> |

| | |
|--|---|
| <p style="text-align: center;">$TK = TR$</p> $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ <p>Substitute $b = 12a - 29$ [from 4.4.4]</p> $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $(a-3)^2 + 144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ <p style="margin-left: 100px;">$a = 2$ or 4</p> $\therefore b = 12(2) - 29 \quad \text{or} \quad b = 12(4) - 29$ $\quad \quad \quad = -5 \quad \quad \quad = 19$ <p>$\therefore K(2; -5)$</p> <p>OR/OF</p> $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = 145 + 145$ $(a-15)^2 + (12a-29-6)^2 = 290$ $(a-15)^2 + (12a-35)^2 = 290$ $a^2 - 30a + 225 + 144a^2 - 840a + 1225 = 290$ $145a^2 - 870a + 1160 = 0$ $a^2 - 6a + 8 = 0$ $\therefore (a-2)(a-4) = 0$ <p>$a = 2$ or $a = 4$</p> $\therefore b = 12(2) - 29 \quad \text{or} \quad b = 12(4) - 29$ $\quad \quad \quad = -5 \quad \quad \quad = 19$ <p>$K(2; -5)$</p> | <p>✓ substitution into distance formula</p> <p>✓ substitution of $b = 12a - 29$</p> <p>✓ $(a-3)^2 = 1$</p> <p>✓ ± 1</p> <p>✓ values of a</p> <p>✓ value of b (6)</p> <p>✓ substitution</p> <p>✓ substitution of $b = 12a - 29$</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ values of a</p> <p>✓ value of b (6)</p> |
| | [23] |

QUESTION/VRAAG 5

| | | |
|--------------|--|---|
| <p>5.1.1</p> | $\sin 196^\circ = -\sin 16^\circ$ $= -p$ | <p>✓ reduction ✓ answer</p> <p>(2)</p> |
| <p>5.1.2</p> | $\cos 16^\circ = \sqrt{1 - \sin^2 16^\circ}$ $= \sqrt{1 - p^2}$ <p>OR/OF</p> $x^2 + p^2 = 1$ $x = \sqrt{1 - p^2}$ $\therefore \cos 16^\circ = \frac{\sqrt{1 - p^2}}{1} = \sqrt{1 - p^2}$  | <p>✓ statement ✓ answer</p> <p>(2)</p> <p>✓ x in terms of p</p> <p>✓ answer</p> <p>(2)</p> |
| <p>5.2</p> | $\sin(A + B) = \cos[90^\circ - (A + B)]$ $= \cos[(90^\circ - A) - B]$ $= \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$ $= \sin A \cos B + \cos A \sin B$ | <p>✓ co-ratio ✓ correct form ✓ expansion</p> <p>(3)</p> |
| <p>5.3</p> | $\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$ $= \frac{\sqrt{\sin^2 2A}}{\cos A \cdot (-\sin A)}$ $= \frac{\sin 2A}{\cos A \cdot (-\sin A)}$ $= \frac{2 \sin A \cos A}{\cos A \cdot (-\sin A)}$ $= -2$ <p>OR/OF</p> $\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cos(90^\circ + A)} = \frac{\sqrt{1 - (2\cos^2 A - 1)^2}}{\cos A \cdot -\sin A}$ $= \frac{\sqrt{1 - (4\cos^4 A - 4\cos^2 A + 1)}}{\cos A \cdot -\sin A} = \frac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A \cdot -\sin A}$ $= \frac{\sqrt{4\cos^2 A(1 - \cos^2 A)}}{\cos A \cdot -\sin A} = \frac{\sqrt{4\cos^2 A \sin^2 A}}{\cos A \cdot -\sin A}$ $= \frac{2\cos A \cdot \sin A}{\cos A \cdot -\sin A}$ $= -2$ <p>OR/OF</p> | <p>✓ $\sqrt{\sin^2 2A}$ ✓ $\cos A$ ✓ $-\sin A$</p> <p>✓ $2\sin A \cos A$</p> <p>✓ answer</p> <p>(5)</p> <p>✓ $2\cos^2 A - 1$ ✓ $\cos A$ ✓ $-\sin A$</p> <p>✓ identity</p> <p>✓ answer</p> <p>(5)</p> |

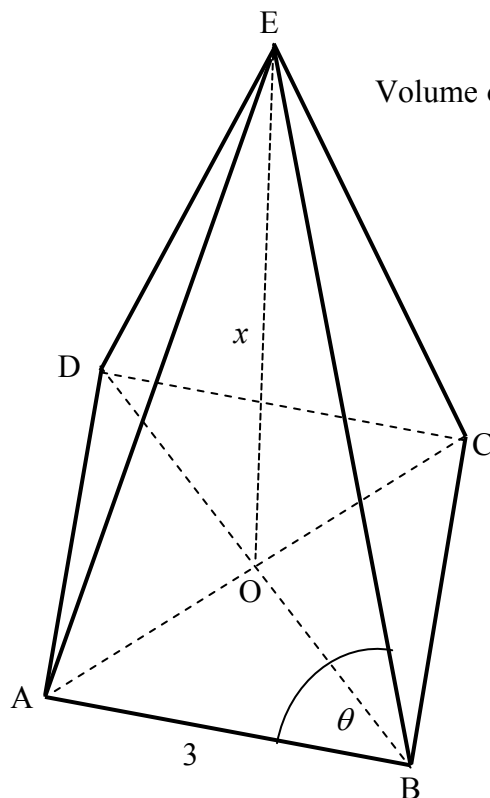
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|--------------|---|--|
| | $\frac{\sqrt{1 - (1 - 2\sin^2 A)^2}}{\cos A - \sin A}$ $= \frac{\sqrt{1 - (1 - 4\sin^2 A + 4\sin^2 A)}}{\cos A - \sin A}$ $= \frac{\sqrt{4\sin^2 A(1 - \sin^2 A)}}{\cos A - \sin A}$ $= \frac{2\sin A \sqrt{\cos^2 A}}{\cos A - \sin A}$ $= -2$ | <p>✓ $1 - 2\sin^2 A$ ✓ $\cos A$ ✓ $-\sin A$</p> <p>✓ identity ✓ answer</p> <p>(5)</p> |
| <p>5.4.1</p> | $\cos 2B = \frac{3}{5}$ $2\cos^2 B - 1 = \frac{3}{5}$ $\cos^2 B = \frac{4}{5}$ $\therefore \cos B = \sqrt{\frac{4}{5}} \text{ or } \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5} \quad [0^\circ \leq B \leq 90^\circ]$ <p>OR/OF</p> $\cos B = \frac{\sqrt{\cos 2B + 1}}{2}$ $= \frac{\sqrt{\frac{3}{5} + 1}}{2}$ $= \frac{2\sqrt{5}}{5}$ | <p>✓ identity ✓ value of $\cos^2 B$ ✓ answer</p> <p>(3)</p> <p>✓ $= \frac{\sqrt{\cos 2B + 1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer</p> <p>(3)</p> |
| <p>5.4.2</p> | $\sin^2 B = 1 - \cos^2 B$ $= 1 - \left(\frac{2}{\sqrt{5}}\right)^2$ $= \frac{1}{5} \quad \therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$ <p>OR/OF</p> $(2)^2 + y^2 = (\sqrt{5})^2$ $4 + y^2 = 5$ $y^2 = 1$ $y = 1$ $\therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$  | <p>✓ $\sin^2 B = \frac{1}{5}$ ✓ answer</p> <p>(2)</p> <p>✓ $y = 1$ ✓ answer</p> <p>(2)</p> |

| | | |
|--------------|---|---|
| | <p>OR/OF</p> $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$ | <p>✓ $\sin^2 B = \frac{1}{5}$</p> <p>✓ answer</p> <p>(2)</p> |
| <p>5.4.3</p> | <p>$\cos(B + 45^\circ) = \cos B \cdot \cos 45^\circ - \sin B \cdot \sin 45^\circ$</p> $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}} \text{ or } \frac{\sqrt{10}}{10}$ <p>OR/OF</p> <p>$\cos(B + 45^\circ) = \cos B \cdot \cos 45^\circ - \sin B \cdot \sin 45^\circ$</p> $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}} \text{ or } \frac{\sqrt{10}}{10}$ | <p>✓ expansion</p> <p>✓ $\left(\frac{1}{\sqrt{2}}\right)$</p> <p>✓ $\left(\frac{2}{\sqrt{5}}\right) \& \left(\frac{1}{\sqrt{5}}\right)$</p> <p>✓ answer</p> <p>(4)</p> <p>✓ expansion</p> <p>✓ $\left(\frac{1}{\sqrt{2}}\right)$</p> <p>✓ $\left(\frac{2}{\sqrt{5}}\right) \& \left(\frac{1}{\sqrt{5}}\right)$</p> <p>✓ answer</p> <p>(4)</p> |
| | | <p>[21]</p> |

QUESTION/VRAAG 6

| | | |
|------------|---|--|
| <p>6.1</p> | | <p>✓ x- intercepts/ afsnitte</p> <p>✓ y- intercept/ afsnit</p> <p>✓ turning pts/ draaipte</p> <p>(3)</p> |
| <p>6.2</p> | <p>$f(x) - 3 = 2 \sin 2x - 3$ \therefore maximum value = $2 - 3 = -1$</p> | <p>✓ ✓ answer</p> <p>(2)</p> |
| <p>6.3</p> | <p>$2 \sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $ref\angle = 26,57^\circ$ $2x = 153,43^\circ + k \cdot 180^\circ; k \in Z$ $x = 76,72^\circ + k \cdot 90^\circ; k \in Z$ or $x = -13,28^\circ + k \cdot 90^\circ; k \in Z$</p> <p>OR/OF</p> <p>$2 \sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $ref\angle = 26,57^\circ$ $2x = 153,43^\circ + k \cdot 360^\circ$ or $333,43^\circ + k \cdot 360^\circ; k \in Z$ $x = 76,72^\circ + k \cdot 180^\circ$ or $166,72^\circ + k \cdot 180^\circ; k \in Z$</p> | <p>✓ $\tan 2x = -\frac{1}{2}$</p> <p>✓ $2x = 153,43^\circ$ or $-26,56^\circ$</p> <p>✓ $76,72^\circ$ or $-13,28^\circ$</p> <p>✓ $k \cdot 90^\circ; k \in Z$</p> <p>(4)</p> <p>✓ $\tan 2x = -\frac{1}{2}$</p> <p>✓ $2x = 153,43^\circ$ & $333,43^\circ$</p> <p>✓ $76,72^\circ$ & $166,72^\circ$</p> <p>✓ $k \cdot 180^\circ; k \in Z$</p> <p>(4)</p> |
| <p>6.4</p> | <p>$x \in (-103,28^\circ; -13,28^\circ)$</p> <p>OR/OF</p> <p>$-103,28^\circ < x < -13,28^\circ$</p> | <p>✓ ✓ values ✓ notation</p> <p>(3)</p> <p>✓ ✓ values ✓ notation</p> <p>(3)</p> <p>[12]</p> |

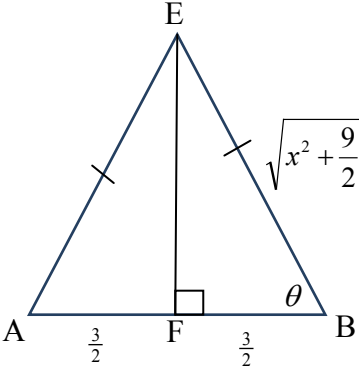
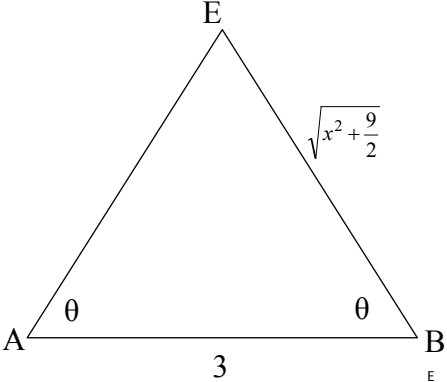
QUESTION/VRAAG 7



Volume of pyramid = $\frac{1}{3}$ (area of base) \times (\perp height)

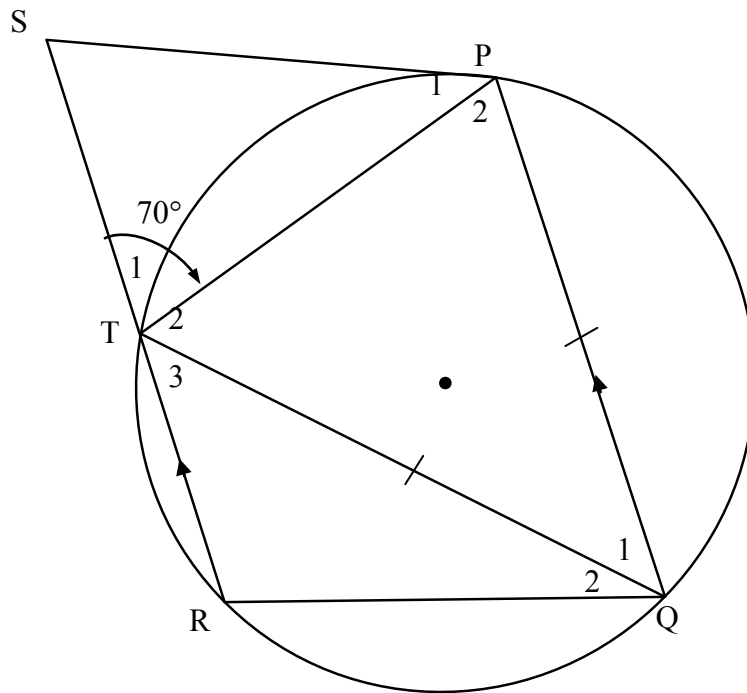
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| 7.1 | $DB^2 = 3^2 + 3^2 \quad \text{[Theorem of Pyth]}$ $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2} DB = \frac{\sqrt{18}}{2} \text{ or } \frac{3}{\sqrt{2}} \text{ or } \frac{3\sqrt{2}}{2} \text{ or } 2,12$ <p>OR/OF</p> $\sin 45^\circ = \frac{OB}{3}$ $OB = 3 \sin 45^\circ$ $OB = \frac{3\sqrt{2}}{2} \text{ or } \frac{3}{\sqrt{2}} \text{ or } 2,12$ <p>OF/OR</p> $\cos 45^\circ = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}} \text{ or } \frac{3\sqrt{2}}{2} \text{ or } 2,12$ | <ul style="list-style-type: none"> ✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3) |
|-----|--|---|

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| | <p>OR/OF $\hat{A}OB = 90^\circ$ (diagonals bisect \perp) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12</p> | <p>✓ $OB = OA$ ✓ Pyth ✓ answer (3)</p> |
| <p>7.2</p> | <p>$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB \cdot EB \cos \theta$ $\cos \theta = \frac{AB^2 + EB^2 - AE^2}{2AB \cdot EB} = \frac{AB^2}{2AB \cdot EB}$ [EB = AE] $\cos \theta = \frac{AB}{2EB}$ $\cos \theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$</p> <p>OR/OF $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB \cdot EB \cos \theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right) \cdot \cos \theta$ $\cos \theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$</p> | <p>✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ $\cos \theta$ as subject ✓ simplification (5)</p> <p style="text-align: right;">s</p> <p>✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ substituting ✓ $\cos \theta$ as subject (5)</p> |

| | | |
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| | <p>OR/OF $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos \theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$</p>  <p>OR/OF $\hat{E} = 180^\circ - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin \theta}$ $\therefore \frac{3}{2 \sin \theta \cos \theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin \theta}$ $\therefore \frac{3}{2 \cos \theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos \theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$</p>  | <p>✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution</p> <p>(5)</p> <p>✓ $\hat{E} = 180^\circ - 2\theta$ ✓ $\sin E = \sin 2\theta$</p> <p>✓ subst into sine rule ✓ diagram ✓ $2 \sin \theta \cos \theta$</p> <p>(5)</p> |
| <p>7.3</p> | <p>Volume = $\frac{1}{3}$(area of base) \times (\perp height) $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos \theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73,97^\circ$</p> | <p>✓ substitution ✓ x-value ✓ substitution ✓ answer</p> <p>(4) [12]</p> |

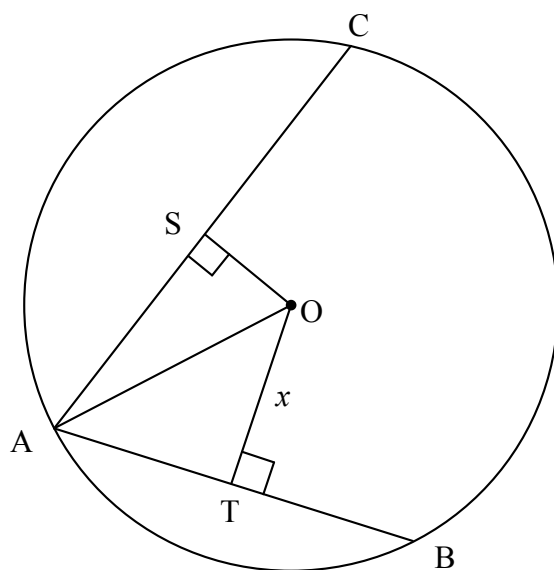
QUESTION/VRAAG 8

8.1



| | | |
|----------|--|----------------------------|
| 8.1.1 | Alternate angles / <i>verwiss hoeke</i> , $PQ \parallel SR$ | ✓ R (1) |
| 8.1.2(a) | $\hat{T}_2 = 70^\circ$ [∠s opp = sides/∠e teenoor = sye] $\therefore \hat{Q}_1 = 180^\circ - 2(70^\circ)$ [∠s/e Δ = 180°] $= 40^\circ$ | ✓ S ✓ R ✓ answer (3) |
| 8.1.2(b) | $\hat{P}_1 = 40^\circ$ [tangent chord th/raakl-koordst] | ✓ S ✓ R (2) |

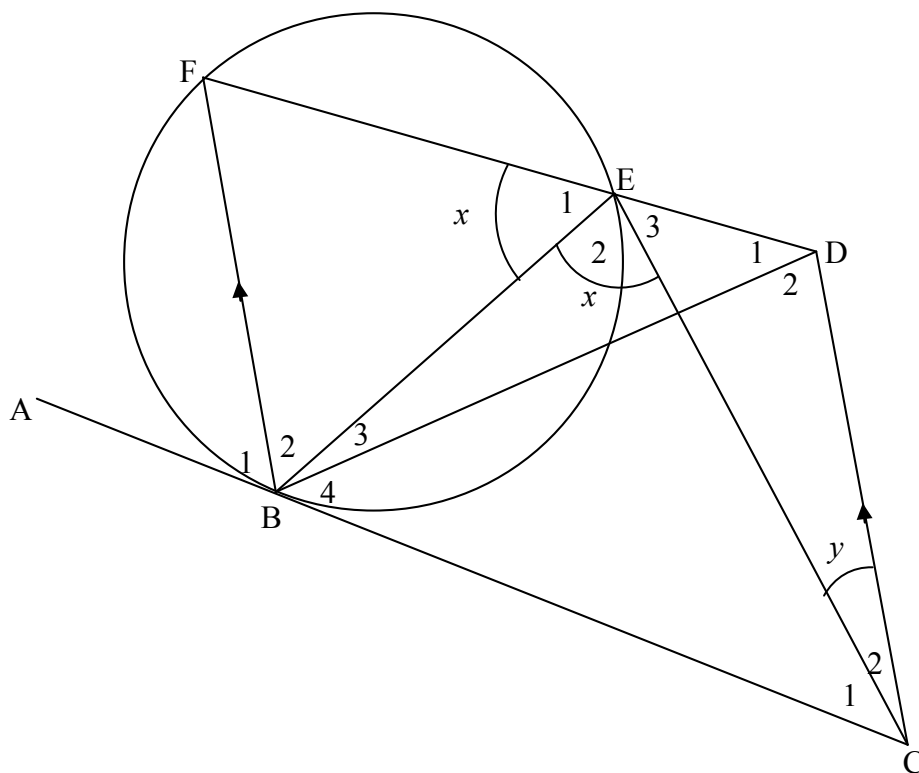
8.2



| | | |
|-------|--|---|
| 8.2.1 | AT = 20 [line from centre \perp to chord/lyn vanaf midpt \perp koord] | \checkmark S (1) |
| 8.2.2 | $AO^2 = OS^2 + AS^2 \quad \text{[Pyth : } \Delta AOS]$ $OT^2 + AT^2 = OS^2 + AS^2 \quad \text{[Pyth : } \Delta AOT]$ <p>But AS = 24 [line from centre \perp to chord/lyn vanaf midpt \perp koord]</p> $OT^2 + 400 = \left(\frac{7}{15} OT\right)^2 + 576$ $176 = \frac{176}{225} OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ <p>OR/OF Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore OA = 25$</p> <p>OR/OF</p> | \checkmark equating \checkmark AS = 24 \checkmark substitution $OS = \frac{7}{15} OT$ \checkmark OT \checkmark radius (5) $\checkmark\checkmark$ testing in ΔAOT $\checkmark\checkmark$ testing in ΔAOS \checkmark conclusion (5) |

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| $AO^2 = OS^2 + AS^2 \quad [\text{Pyth : } \Delta AOS]$ $OT^2 + AT^2 = OS^2 + AS^2 \quad [\text{Pyth : } \Delta AOT]$ <p>Let $OT = 15x$. Then $OS = 7x$ But $AS = 24$ [line from centre \perp to chord/lyn vanaf midpt \perp koord]</p> $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ <p>OR/OF $AS = 24$ [line from centre \perp to chord/lyn vanaf midpt \perp koord]</p> $AO^2 = OS^2 + AS^2 \quad [\text{Pyth : } \Delta AOS]$ $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2 \quad [\text{Pyth : } \Delta AOT]$ $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$ | <p>✓ equating</p> <p>✓ $AS = 24$ ✓ substitution</p> <p>✓ $x = 1$</p> <p>✓ radius (5)</p> <p>✓ $AS = 24$</p> <p>✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth</p> <p>✓ radius (5) [12]</p> |
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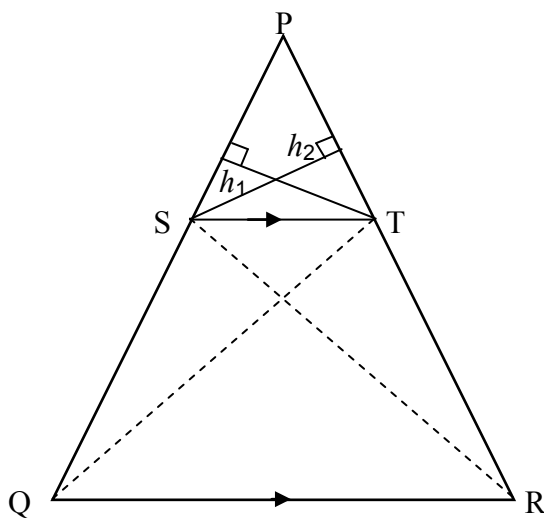
QUESTION/VRAAG 9



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| 9.1.1 | tangent chord theorem/raaklyn-koordstelling | ✓ R | (1) |
| 9.1.2 | corresponding/ooreenkomstige \angle s/e; $FB \parallel DC$ | ✓ R | (1) |
| 9.2 | $\hat{E}_1 = \hat{C}_D$ $\therefore BCDE = \text{cyclic quad}$ [converse ext \angle cyc quad/omgek: buite \angle kdvh] | ✓ S ✓ R | (2) |
| 9.3 | $\hat{D}_2 = \hat{E}_2$ [\angle s in the same segment/ \angle e in dies segment] $\hat{D}_2 = \hat{F}_B D$ [alt \angle s, $BF \parallel CD$ /verwiss \angle e, $BF \parallel CD$] | ✓ S ✓ S | (2) |
| 9.4 | $\hat{B}_3 = y$ OR $\hat{B}_3 = \hat{C}_2$ [\angle s in the same segment/ \angle e in dies segment] $\hat{B}_2 = x - y$ OR $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ OR/OF In $\triangle BFE$ and $\triangle BEC$ $\hat{E}_1 = \hat{E}_2$ [= x] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \triangle BFE \parallel \triangle CBE$ [\angle, \angle, \angle] $\therefore \hat{B}_2 = \hat{C}_1$ | ✓ S ✓ S ✓ S ✓ identifying Δ 's ✓ S ✓ S | (3) [9] |

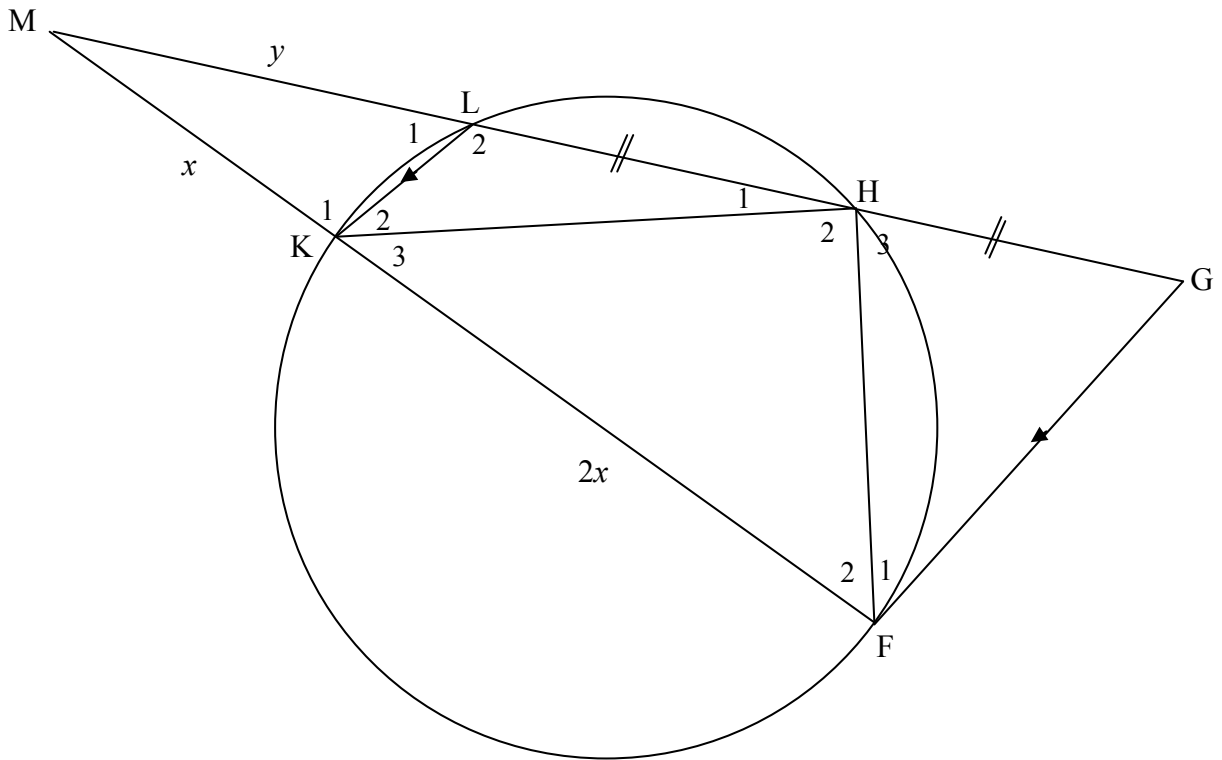
QUESTION/VRAAG 10

10.1



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| 10.1 | <p>Constr : Join S to R and T to Q and draw h_1 from S \perp PT and h_2 from T \perp PS/ <i>Verbind SR en TQ en trek h_1 van S \perp PT en h_2 van T \perp PS]</i></p> <p>Proof :</p> $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \frac{PS}{SQ} \quad \text{equal altitudes}$ $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2} PT \times h_1}{\frac{1}{2} TR \times h_1} = \frac{PT}{TR} \quad \text{equal altitudes}$ <p>area $\Delta PST = \text{area } \Delta PST$ [common] But area $\Delta QST = \text{area } \Delta STR$ [same base, height; ST \parallel QR]</p> $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$ | <p>✓ constr/konstruksie</p> <p>✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p style="text-align: right;">(6)</p> |
|------|--|--|

10.2



| | | |
|-----------|--|---|
| 10.2.1 | Corresponding/Ooreenkomstige \angle s/e; $GF \parallel LK$ | \checkmark R (1) |
| 10.2.2(a) | $\frac{GL}{LM} = \frac{FK}{KM}$ OR $\frac{GL}{y} = \frac{2x}{x}$ [prop theorem/eweredighst; $GF \parallel LK$] $\frac{2GH}{y} = \frac{2x}{x}$ [LH = HG] $\therefore GH = y$ | \checkmark S \checkmark R \checkmark $GL = 2GH$ (3) |

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| <p>10.2.2(b)</p> | <p>$\tilde{K}_1 = \hat{G}\hat{F}M$ $L\hat{K}M$ or $\tilde{K}_1 = M\hat{H}F$ $M\hat{H}F = \hat{G}\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ $M\hat{H}F = \hat{G}\hat{F}M$ $\therefore \Delta MFH \parallel \Delta MGF$ OR/OR $\tilde{K}_1 = \hat{G}\hat{F}M$ $L\hat{K}M$ or $\tilde{K}_1 = M\hat{H}F$ $M\hat{H}F = \hat{G}\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ $M\hat{H}F = \hat{G}\hat{F}M$ $\hat{F}_2 = \hat{G}$ $\therefore \Delta MFH \parallel \Delta MGF$</p> | <p>[corresponding/ooreenkomst \angle s; $GF \parallel LK$] [ext \angle cyclic quad/buite\anglekoordevh] [common/gemeen] [proven/bewys] [$\angle\angle\angle$] [corresponding/ooreenkomst \angle s; $GF \parallel LK$] [ext \angle cyclic quad/buite\anglekoordevh] [common/gemeen] [proven/bewys] [\angles of $\Delta = 180^\circ$]</p> | <p>$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark R$ (5) $\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark S$ (5)</p> |
| <p>10.2.2(c)</p> | <p>$\therefore \frac{GF}{FH} = \frac{MF}{MH}$ $= \frac{3x}{2y}$</p> | <p>[$\parallel \Delta$s]</p> | <p>$\checkmark S \checkmark R$ (2)</p> |
| <p>10.2.3</p> | <p>$\frac{MF}{MH} = \frac{MG}{MF}$ $\frac{3x}{2y} = \frac{3y}{3x}$ $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$</p> | <p>[$\parallel \Delta$s] [from 10.2.2(c)]</p> | <p>$\checkmark S$ \checkmark substitution \checkmark simplification (3) [20]</p> |
| <p>TOTAL MARKS</p> | | | <p>150</p> |