NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

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SECTION A

QUESTION 1

(a) $A(0\,;3)$

(b) $-2x+3=3x-7$
$-5x=-10$
$x=2$
$y=-1$

(c) $d=\sqrt{(3-(-1))^2+(0-2)^2}$
$d=\sqrt{20}$

(d) $(x-2)^2+(y+1)^2=20$

(e) $(4;-5)$

(f) $0=-2x+3$
$x=\dfrac{3}{2}$
$0=3\left(\dfrac{3}{2}\right)+c$
$y=3x-\dfrac{9}{2}$

QUESTION 2

(a) (1) Method of finding third side
$\sqrt{1-m^2}$

(2) $2\cos^2 25°-1$ or $\cos^2 25°-\sin^2 25°$
$=2m^2-1$ or $m^2-(\sqrt{1-m^2})^2$

(3) $\cos 30°\cos 25°-\sin 30°\sin 25°$

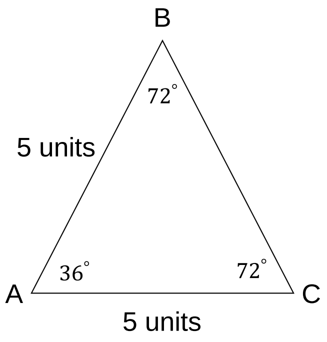
$\dfrac{\sqrt{3}\,m}{2}-\dfrac{\sqrt{1-m^2}}{2}$

(b) $\frac{\sin(21^\circ + w)}{\sin(w + 21^\circ)} - \frac{\sin^2 \beta}{\cos^2 \beta}.\cos^2 \beta$

$= 1 - \sin^2 \beta$

$= \cos^2 \beta$

(c)



$\text{Area}\,\Delta ABC = \frac{1}{2} \times 5 \times 5 \times \sin 36^\circ$

$\text{Area}\ \Delta ABC = 7{,}3\ \text{units}^2$

QUESTION 3

(a) p = 2

(b) 180°

(c) $3\sin 2x = 2\sin x$

$6\sin x \cos x = 2\sin x$

$2\sin x(3\cos x - 1) = 0$

$\cos x = \frac{1}{3}$

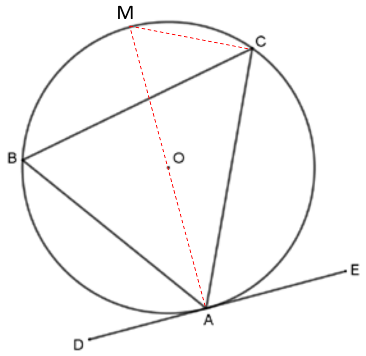
$x = 70{,}5^\circ$

$B(289{,}5^\circ; -1{,}9)$

(d) k > 3

QUESTION 4

(a)

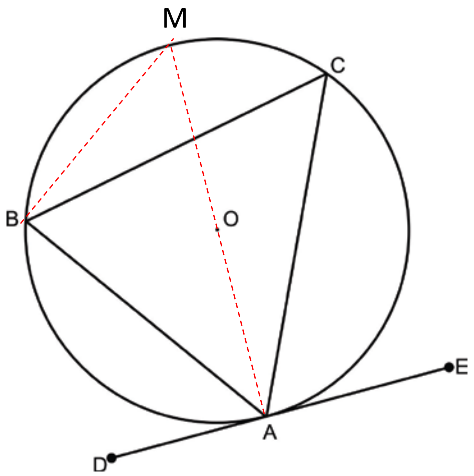


Construction AM and MC

$E\hat{A}M = 90°$ (Line from centre perpendicular to tangent)
$M\hat{C}A = 90°$ (Angle in semi-circle)
$C\hat{M}A + M\hat{A}C = 90°$ (Angles in a triangle)

Therefore
$A\hat{M}C = E\hat{A}C$
but
$A\hat{M}C = A\hat{B}C$ (Angles in same segment)
Hence
$E\hat{A}C = A\hat{B}C$

Alternate solution (there are a number of others)



Construction AM and MB

$M\hat{B}A = 90°$ (Angle in semi-circle)
$M\hat{A}E = 90°$ (Line from centre perpendicular to tangent)
$M\hat{B}C = M\hat{A}C$ (Angles in same segment)
$C\hat{B}A = C\hat{A}E$

(b) $\hat{F}_1 = 55°$ (Tan chord theorem)
$\hat{F}_2 = 35°$ (Angles in semi-circle)
$\hat{E}_1 = 35°$ (Angles in same segment)

(c) $\hat{C}_1 = 110°$ (CF = CG radii; Isos triangle)
$\hat{D}_1 = 80°$ (Angle @ centre)
$\hat{D}_3 = 80°$ (Vert opp)
$\hat{D}_3 + \hat{B} = 180°$

Therefore DHBJ is a cyclic quad (Converse: Opp. angles of a cyclic quad are supplementary)

Alternate solution

$\hat{C}_1 = 110°$ (CF = CG radii; Isos triangle)
$\hat{D}_1 = 80°$ (Angle @ centre)
$\hat{D}_2 = 100°$ (Angles on a straight line)
$\hat{D}_2 = \hat{B}$

Therefore DHBJ is a cyclic quad (Converse: Exterior angle of a cyclic quad is equal to the interior opposite angle)

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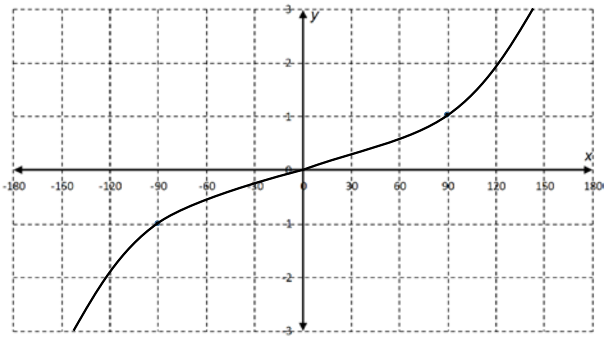
QUESTION 5

(a) (1) $x = 180° + k.360$

(2) Shape of graph

Points on $(-90°; -1)$ and $(90°; 1)$

Understanding of Asymptotes at $-180°$ and $180°$



(b) $= \dfrac{\cos\theta\sin^2\theta + \cos^3\theta}{\cos\theta} - 2\sin^2\theta$

$= \dfrac{\cos\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta} - 2\sin^2\theta$

$= 1 - 2\sin^2\theta$ (5)

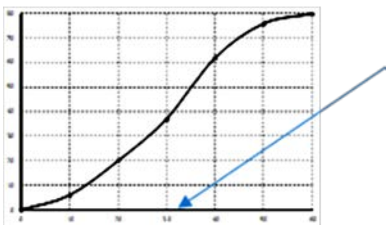
$\cos 2\theta$

$= 1 - 2\sin^2\theta$

QUESTION 6

(a) 16

(b) (1) Look at the diagram



(2) IQR = 38 – 20 = 18

(c) (1) A = –2,6 and B = 2

(2) Extrapolation or prediction is way outside HV used

(d) (1)

(2) The values of those to the right of median are more spread out thus the mean is pulled to the right of the median.

(e) Yes; it needs a service as one standard deviation from the mean shows that a large percentage of the logs are between 9,2 and 10,8 metres. This shows that the machine is inaccurate.

SECTION B

QUESTION 7

(a) –0,8

(b) (1) Get closer to –1 or decrease

(2) Decrease or get steeper

(3) A person who plays professional sport and studies via the internet

(c) (1) More of the data is closer to the mean
OR 4 low and 3 high hence further apart

(2) Mean would decrease and standard deviation will decrease

(3) Decrease the price on any day from Monday to Saturday, increase in sales and data will be more dispersed **OR** any intervention that would increase the sales on a day between Monday and Saturday.

QUESTION 8

(a) (1) $\frac{CN}{NE} = \frac{DG}{GE} = \frac{2}{3}$

$DG = GK$

Therefore

$\frac{EK}{KG} = \frac{1}{2}$

(2) $\frac{DH}{HF} = \frac{4}{1}$

$DE = 5m$ and $DF = 5k$

$$\frac{\frac{1}{2} \times 2m \times 4k \times \sin\hat{D}}{\frac{1}{2} \times 5m \times 5k \times \sin\hat{D}}$$

$$= \frac{8}{25}$$

(b) (1) $\hat{E}_2 = x$ (Angle at centre)

$\hat{N}_3 = x$ (Alternate angles // lines)

$\hat{G}_1 = x$ (Tan chord theorem)

ALT: $\hat{E}_2 = \hat{N}_3 = \hat{G}_1$ (With the reasons above)

Therefore

$GN = NE$ (Converse: isos triangle)

(2) $G\hat{N}E = 180^\circ - 2x$ (Angles in a triangle)

$\hat{H}_2 = 2x$ (Opp. angles of cyclic quad)

$\hat{E}_1 = \hat{N}_1 = 90^\circ - x$ (Angles in isos triangle)

$\Delta GON ||| \Delta GHE$ (A.A.A)

(3) Working with HE

$\frac{HE}{ON} = \frac{GE}{GN}$ (Similar triangles)

$\frac{HE}{EP} = \frac{HB}{BC}$ (Prop theorem)

QUESTION 9

(a) $\text{FH} = \sqrt{50}$

$\text{HG} = \sqrt{65}$

$10^2 = 50 + 65 - 2 \times \sqrt{50} \times \sqrt{65} \cos \hat{H}$

$\hat{H} = 82,4^\circ$

ALT: Cosine rule for any two sides and angle

Area $\Delta FGH = \frac{1}{2} \times \sqrt{50} \times \sqrt{65} \times \sin 82,4^\circ$

Area $\Delta FGH = 28,3 m^2$

(b) (1) Coordinates of C

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

C(4; 0)

(2) $x^2 - 6x + 9 + y^2 - 4y + 4 = -8 + 13$

$(x-3)^2 + (y-2)^2 = 5$

Centre of circle (3; 2)

Gradient of line $AB \times \frac{1}{2} = -1$

Gradient of line $AB = -2$

$3 = -2(5) + c$

$c = 13$

$y = -2x + 13$

Coordinates of B

$0 = -2x + 13$

$x = 6,5$

$B(6,5;0)$

Therefore CB = 2,5 units

QUESTION 10

(a) $y = \frac{2}{3}x + \frac{8}{3}$

$\tan F\hat{E}A = \frac{2}{3}$ $\quad F\hat{E}A = 33,7°$

$\tan E\hat{F}A = -1$ $\quad E\hat{F}A = 45°$

Therefore
$E\hat{A}F = 101,3°$

(b) $\frac{AF}{\sin 33,7°} = \frac{\sqrt{52}}{\sin 45°}$

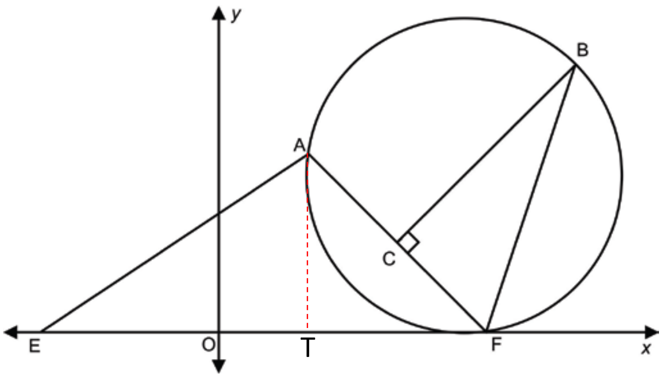
$AF = 5,66$

$CF = 2,83$ (Line from centre perpendicular to chord)

$CB^2 = \sqrt{40}^2 - 2,83^2$

$CB = 5,7$

Alternate solution



$\frac{AT}{\sqrt{52}} = \sin 33,7°$

AT = 4

TF = 4 units (Isos Triangle)

AF = $\sqrt{32}$

$CF = 2,83$ (Line from centre perpendicular to chord)

$CB^2 = \sqrt{40}^2 - 2,83^2$

$CB = 5,7$

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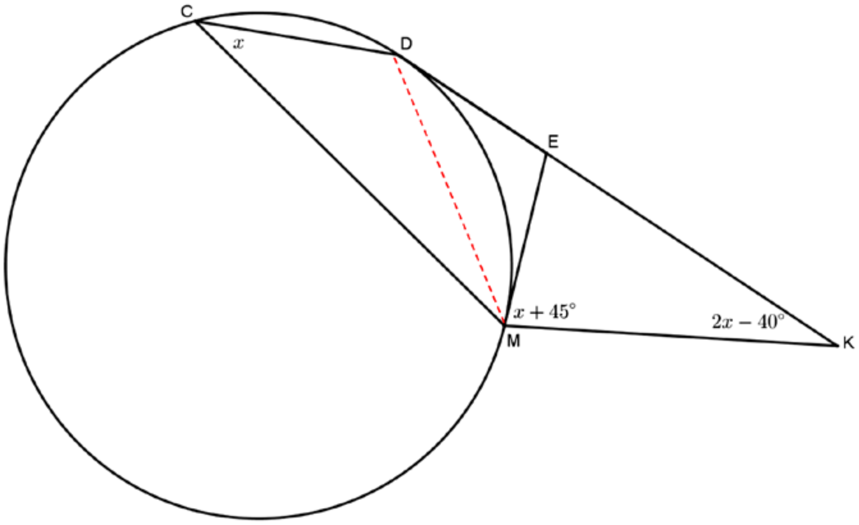
QUESTION 11



Construction MD

$M\hat{D}E = x$ (Tan chord theorem)
$D\hat{M}E = x$ (Tan chord theorem) OR (isos triangle; tangents from common point)

$$D\hat{E}M = 180° - 2x$$
$$180° - 2x = x + 45° + 2x - 40° \quad \text{(Ext angle of triangle)}$$
$$-5x = -175°$$
$$x = 35°$$

QUESTION 12

(a) $5x + 12y = 60$

A(0; 5) and $B(12;0)$

$AB^2 = 5^2 + 12^2$
$AB = 13$

(b) $\Delta ADC : \Delta BCD = 8:18$

$$AC = 13 \times \frac{8}{26}$$

$$AC = 4 \text{ units}$$

$$(5k)^2 + (12k)^2 = 4^2$$

$$169k^2 = 16$$

$$k^2 = \frac{16}{169}$$

$$k = \frac{4}{13}$$

Coordinates of C

$$C\left(\frac{48}{13}; \frac{45}{13}\right)$$

Alternate solution

$\Delta ADC : \Delta BCD = 8:18$

$$AC = 13 \times \frac{8}{26}$$

$$AC = 4 \text{ units}$$

If C(a; b) then:

$\frac{a}{12} = \frac{4}{13}$ (line parallel to side of Δ)

Coordinates of C

$$C\left(\frac{48}{13}; \frac{45}{13}\right)$$

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Alternate solution

$$\Delta ADC : \Delta BCD = 8 : 18$$
$$AC = 13 \times \frac{8}{26}$$
$$AC = 4 \text{ units}$$

$$C\hat{B}O = 22,62^\circ$$
$$\sin 22,62^\circ = \frac{y}{9}$$
$$y = 3,5 \text{units}$$

$$\cos 22,62^\circ = \frac{m}{9}$$

$$m = 8,3$$

$$x = 12 - 8,3 = 3,7$$

$C(3,7;3,5)$

Total: 150 marks

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