basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2-2x=14$ (correct to TWO decimal places) (4)

1.1.3 $2^{x+2}+2^x=20$ (3)

1.2 Solve the following equations simultaneously:

$x=2y+3$
$3x^2-5xy=24+16y$ (6)

1.3 Solve for x: $(x-1)(x-2)<6$ (4)

1.4 The roots of a quadratic equation are: $x=\dfrac{3\pm\sqrt{-k-4}}{2}$

For which values of k are the roots real? (2)

[21]

QUESTION 2

Given the arithmetic series: 2 + 9 + 16 + ... (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251^{st} term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)

[12]

QUESTION 3

3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \ldots$

3.1.1 Write down the value of p. (2)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: 16 ; 4 ; 1

3.2.1 Calculate the value of the 12$^{\text{th}}$ term. (Leave your answer in simplified exponential form.) (3)

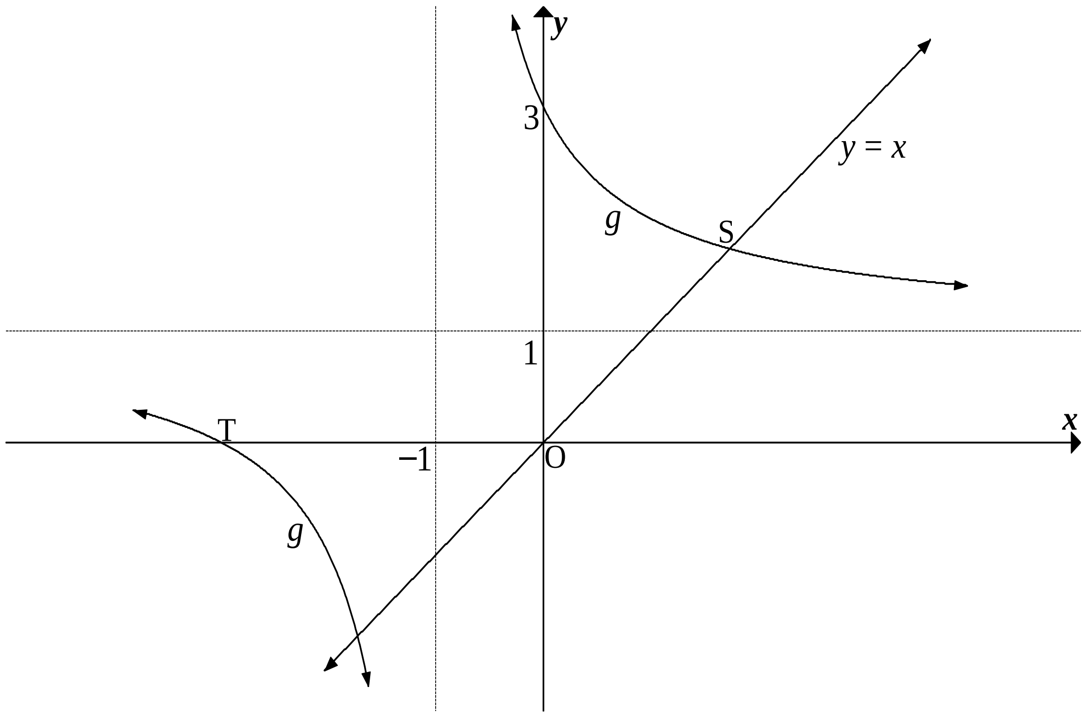
3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)$ ... up to 98 factors. (4)

[19]

QUESTION 4

The diagram below shows the hyperbola g defined by $g(x) = \dfrac{2}{x+p} + q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at (0 ; 3). The line $y = x$ intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of p and q. (2)

4.2 Calculate the x-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x) = g(x+5)$ (1)

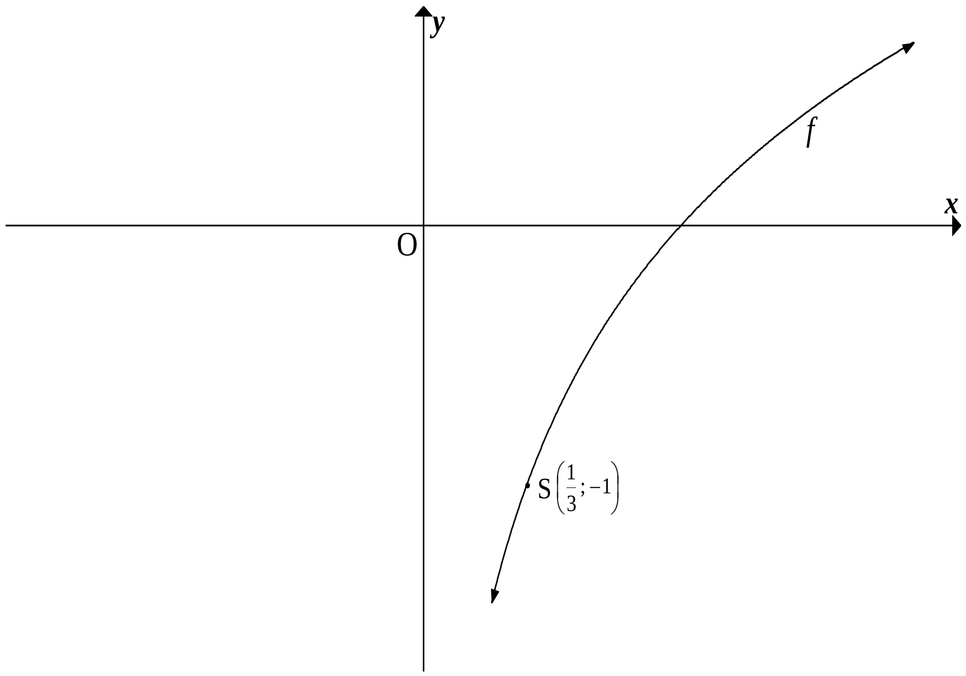
4.4 Calculate the length of OS. (5)

4.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1)

[11]

QUESTION 5

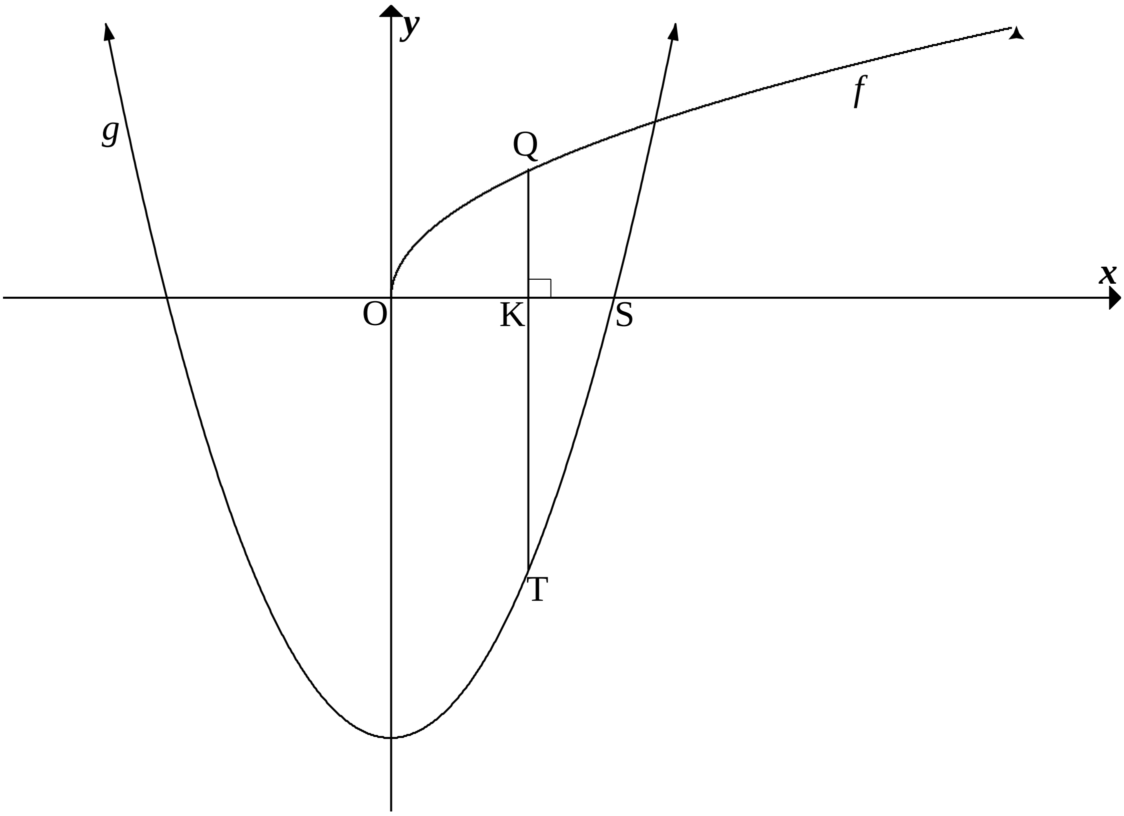
Given: $f(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of f.



5.1	Prove that $a = 3$.	(2)
5.2	Write down the equation of h, the inverse of f, in the form $y = ...$	(2)
5.3	If $g(x) = -f(x)$, determine the equation of g.	(1)
5.4	Write down the domain of g.	(1)
5.5	Determine the values of x for which $f(x) \geq -3$.	(3)
		[9]

QUESTION 6

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



6.1 Determine the x-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3 6.3.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K. (3)

6.3.2 Calculate the maximum length of QT. (6)

[13]

QUESTION 7

7.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation. (3)

7.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

7.2.1 Calculate the value of Samuel's monthly instalment. (4)

7.2.2 Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

7.2.3 Who pays more interest, Samuel or Melissa? Justify your answer. (2)

[13]

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = \left(x^6 - 1\right)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)

[14]

QUESTION 9

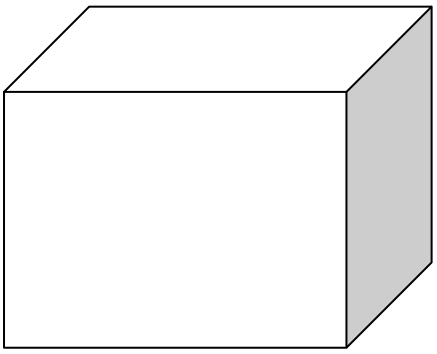
Given: $f(x) = (x+2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

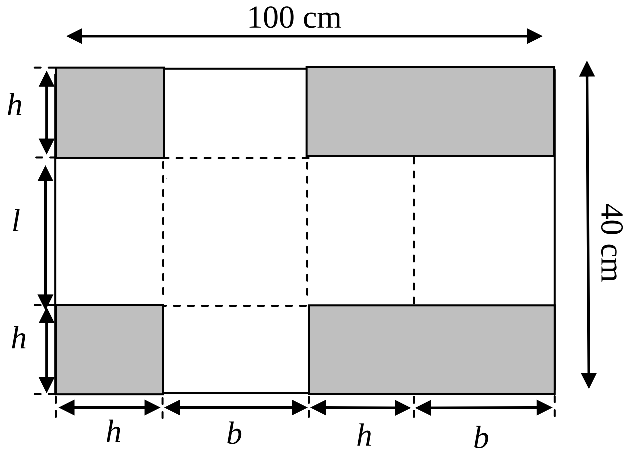
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f'(x) < 0$? (3)
[13]

QUESTION 10





A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)
[9]

QUESTION 11

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

11.1 Determine the probability that a randomly selected staff member:

11.1.1 Is male (1)

11.1.2 Does not prefer visiting a game park (2)

11.2 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)
[7]

QUESTION 12

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 All the letters of the alphabet can be used (2)

12.1.2 The password must start with a 'D' and end with an 'L' (2)

12.2 Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

12.2.1 In how many different ways can ALL the cars be parked? (2)

12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)
[9]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$In\ \Delta ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$