MEASUREMENTS TEST

Grade 10
Mathematics
Marks: **50**
Time: **1 hour**

Name: _____**MEMORANDUM**_____

REFRACTION
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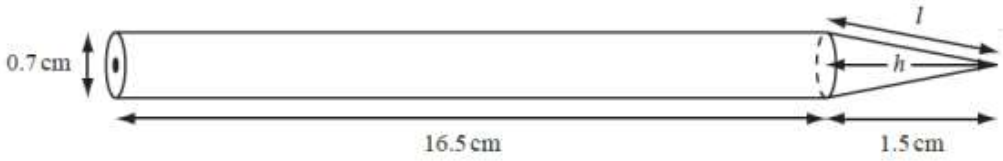
QUESTION 1

The diagram shows a pencil of length 18 cm.

- It is made from a cylinder and a cone.
- The cylinder has a diameter 0,7 cm and length 16,5 cm.
- The cone has diameter 0,7 cm and length 1,5 cm.

$$Volume\ of\ a\ cylinder = \pi r^2 h$$

$$Volume\ of\ a\ cone = \frac{1}{3}\pi r^2 h$$



1.1 Calculate the volume of the pencil. (5) S1112

$V_{cyl} = \pi(0{,}35)^2(16{,}5)$ ✓

$= 6{,}35$ ✓

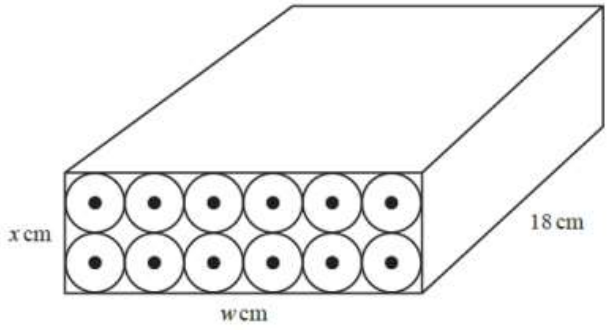
$V_{cone} = \frac{1}{3}\pi(0{,}35)^2(1{,}5)$ ✓

$= 0{,}19$ ✓

$V_{pencil} = 6{,}35 + 0{,}19$

$= 6{,}54\ cm^3$ ✓

1.2 12 of these pencils fit exactly into a rectangular box of length 18 cm, width w cm and height x cm. The pencils are in 2 rows of 6 as shown in the diagram below.



1.2.1 Write down the values of w and x. (2)

$w = 4{,}2\ cm$ ✓

$x = 1{,}4\ cm$ ✓

1.2.2 Calculate the volume of the box. (2) S1111

$V = l \times b \times h$

$= 4{,}2 \times 1{,}4 \times 18$ ✓

$= 105{,}84\ cm^3$ ✓

1.2.3 Calculate the percentage of the volume of the box occupied by the pencils. (2)

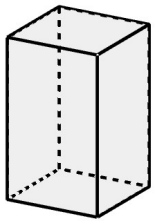
$\frac{6{,}54 \times 12}{105{,}84} \times 100$ ✓

$= 74{,}15\%$ ✓

[11]

QUESTION 2

A mechanic uses 36 000 cm^3 of oil to fill a tank which is in the shape of a rectangular prism.



2.1 By how much will the volume of the tank increase if the length and breadth of the tank is doubled and the height remains the same? (2) S1115

$V = l \times b \times h$

$V_{new} = 2l \times 2b \times h$ ✓

$= 4(l \times b \times h)$

$\therefore V_{new} = 4V$ ✓

2.2 What is the new volume? (1)

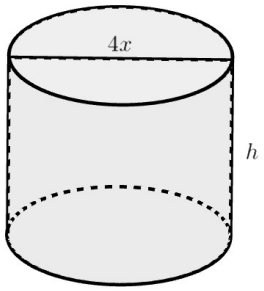
$V_{new} = 4(36\ 000)$

$= 144\ 000\ cm^3$ ✓

[3]

QUESTION 3

The cylinder in the diagram to the right has a diameter of $4x$ units and a height of h. The cylinder is open at the top and the total surface area of the cylinder is $32\pi\ units^2$.



$Total\ surface\ area\ of\ a\ cylinder$ $= 2\pi r^2 + 2\pi rh$

Calculate the height of the cylinder in terms of x. (4) S1114

$r = 2x$ ✓

$TSA = \pi r^2 + 2\pi rh$

$32\pi = \pi(2x)^2 + 2\pi(2x)h$ ✓

$32\pi = 4\pi x^2 + 4\pi xh$

$8 = x^2 + xh$ ✓

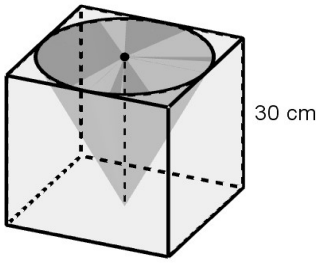
$8 - x^2 = xh$

$h = \frac{8 - x^2}{x}$ ✓

[4]

QUESTION 4

A perfect cone is cut out of a metal cube of sides equal to 30 cm, so that the circumference of the cube's base touches the edges of the cube and the vertex of the cone lies on the cube's base as shown to the right.



Determine the volume of metal left after the cone has been removed. (5) S1116

$V_{cube} = 30 \times 30 \times 30$ ✓

$= 27\ 000$ ✓

$V_{cone} = \frac{1}{3}\pi(15)^2(30)$ ✓

$= 7068{,}58$ ✓

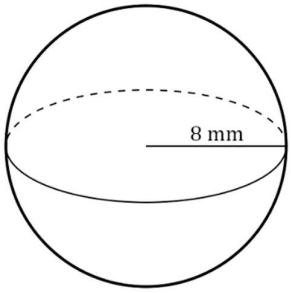
$V = 27\ 000 - 7068{,}58$

$= 19\ 931{,}42\ cm^3$ ✓

[5]

QUESTION 5

A metal ball has a radius of 8 millimetres.



$$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

$$Total\ surface\ area\ of\ a\ sphere = 4\pi r^2$$

5.1 Calculate the volume of metal used to make this ball, correct to TWO decimal places (2) S1112

$V_{sphere} = \frac{4}{3}\pi(8)^3$ ✓

$= 2\ 144{,}66\ mm^3$ ✓

5.2 If the radius is doubled, write down the ratio of the new volume: the original volume. (2) S1115

$V_{new} = \frac{4}{3}\pi(2r)^3$ ✓

$= 8\left(\frac{4}{3}\pi r^3\right)$

$= 8V$

$\therefore 8 : 1$ ✓

5.3 You would like the ball to be silver plated to a thickness of 1 millimetre. What is the volume of the silver required? Give your answer correct to TWO decimal places (2) S1116

$V_{outer} = \frac{4}{3}\pi(9)^3$

$= 3\ 053{,}638$ ✓

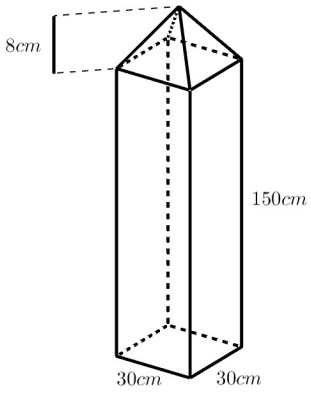
$V_{silver} = 3\ 053{,}638 - 2\ 144{,}66$

$= 908{,}97\ mm^3$ ✓

[6]

QUESTION 6

A concrete gate post compromises a right rectangular prism having a square base and a pyramid at the top, as shown in the diagram on the right. The length of the sides of the base is 30 cm and the height of the rectangular section is 150 cm. The perpendicular height of the pyramid section is 8 cm.



$Volume\ of\ a\ pyramid$

$= \frac{1}{3} area\ of\ base \times height$

$Total\ surface\ area\ of\ pyramid$

$= area\ of\ base + \frac{1}{2}(perimeter\ of\ the\ base \times slant\ height)$

6.1 Calculate the volume of the concrete required to make ONE post. (3) S1111

$V_{pyr} = \frac{1}{3}(30 \times 30) \times 8$

$= 2\ 400$ ✓

$V_{prism} = 30 \times 30 \times 150$

$= 135\ 000$ ✓

$V_{post} = 2\ 400 + 135\ 000$

$= 137\ 400\ cm^3$ ✓

6.2 Calculate the surface area of the pyramid section of the post. (3) S1113

$Slant\ height = \sqrt{15^2 + 8^2}$

$= 17$ ✓

$TSA = \frac{1}{2}(4 \times 30 \times 17)$ ✓

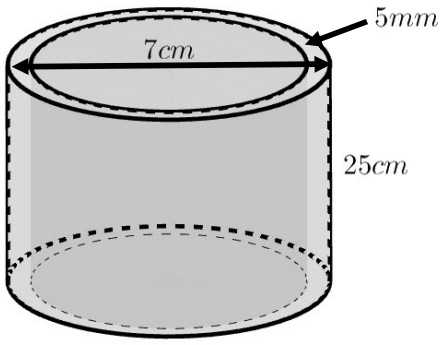
$= 1\ 020\ cm^2$ ✓

[6]

QUESTION 7

A cylindrical flask has a diameter of 7 cm. The metal used to make the flask is 5 mm thick.

$$Volume\ of\ a\ cylinder = \pi r^2 h$$
$$Total\ surface\ area\ of\ a\ cylinder = 2\pi r^2 + 2\pi rh$$



7.1 Show that the radius of the inner flask is 3 cm. (1)

$diameter = 7 - 0{,}5 - 0{,}5 = 6$
$radius = 6 \div 2 = 3\ cm$ ✓

7.2 Calculate the cross-sectional area of the inner flask. (2) S1114

$SA = \pi r^2 + 2\pi rh$
$= \pi(3)^2 + 2\pi(3)(25)$ ✓
$= 499{,}51\ cm^2$ ✓

7.3 If the flask is 25 cm tall, calculate the volume of liquid that the flask can hold. (2) S1112

$V = \pi(3)^2(25)$ ✓
$= 706{,}86\ cm^3$ ✓

7.4 Calculate the surface area of the flask without a lid. (3) S1114

$SA = \pi r^2 + 2\pi rh$ ✓
$= \pi(3{,}5)^2 + 2\pi(3{,}5)(25)$ ✓
$= 588{,}26\ cm^2$ ✓

7.5 By what factor would the volume of the flask be increased if the radius doubled? (2) S1115

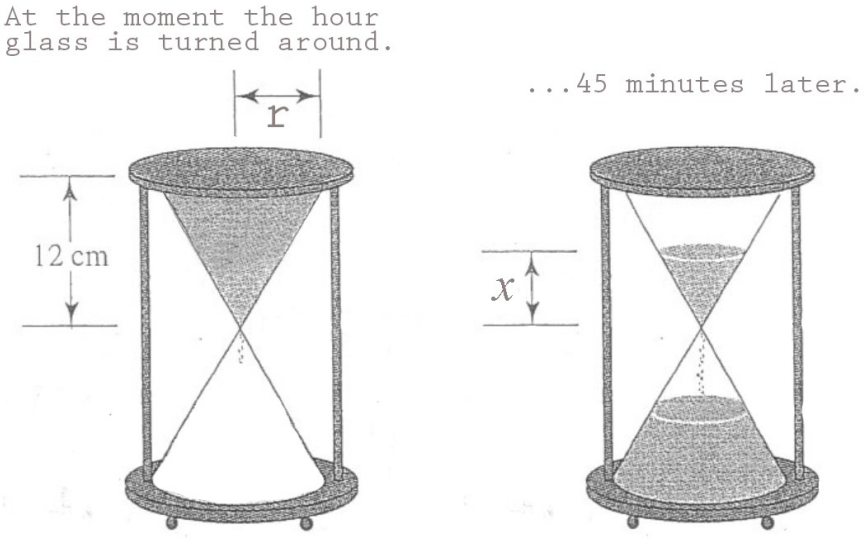
$V_{new} = \pi(2r)^2 h$ ✓
$= 4\pi r^2 h$

$V_{new} = 4V$ ✓

[10]

QUESTION 8

An hourglass is made up of 2 cones. The moment it is turned so that all the sand is in the top cone the height of the cone that the sand forms is 12 cm and THE RADIUS IS ALWAYS HALF OF THE HEIGHT.



Calculate the height, x, of the 'sand cone' after 45 minutes. (5) Level 4

$r_0 = 12 \div 2 = 6$ ✓

$V_0 = \frac{1}{3}\pi(6)^2(12)$

$= 144\pi$ ✓

$V_{45} = 144\pi \times \frac{15}{60}$

$= 36\pi$ ✓

$V_{45} = \frac{1}{3}\pi \times \left(\frac{x}{2}\right)^2 \times x$ ✓

$36\pi = \frac{1}{3}\pi \times \frac{x^3}{4}$

$x^3 = 432$

$x = 7{,}56\ cm$ ✓

[5]

Total: 50 Marks